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THE GLUON IMPACT FACTORS *

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Abstract

We calculate in the next-to-leading approximation the non-forward gluon impact factors for arbitrary color state in the t -channel. In the case of the octet state we check the so-called "second bootstrap condition" for the gluon Reggeization in QCD, using the integral representation for the impact factors. The condition is fulfilled in the general case of an arbitrary space-time dimension and massive quark flavors for both helicity conserving and non-conserving parts.

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1 Introduction

The BFKL equation [1] is widely discussed now, because it can enlighten an important question of elementary particle physics such as the theoretical description of QCD semi-hard processes. It gets a special importance due to the present experimental investigation of the deep inelastic electron-proton scattering at HERA (see, for example, [2]) in the region of small values of the Bjorken variable x . This equation was derived more than twenty years ago in the leading logarithmic approximation (LLA) [1], where all the terms of the type $\alpha_s^n \ln^n(1/x)$ are summed up. Recently, the radiative corrections to the equation were calculated [3]-[8] and the explicit form of the kernel of the equation in the next-to-leading approximation (NLA) became known [9, 10] for the case of forward scattering. The large size of the corrections induced a number of subsequent publications (see, for instance, [11]).

In the BFKL approach the high energy scattering amplitudes are given as the convolution (see Eq.(2.5) below) of the Green function for two interacting Reggeized gluons with the impact factors of the colliding particles [1, 9, 12, 13, 14]. While the Green function is determined by the kernel of the BFKL equation, the impact factors must be evaluated separately. In some cases such as, for instance, the impact factors of strongly-virtual photons or hard mesons, this can be done in perturbation theory, while in the general non-perturbative case there is need of new ideas for the evaluation.

This paper is devoted to the calculation of the NLA non-forward gluon impact factors with arbitrary color state in the t -channel. Although they are not directly connected with observable cross sections, their knowledge is very important for the BFKL theory for two reasons. Firstly, these impact factors can be used for the NLA calculation of scattering amplitudes of partons in the BFKL approach. The second reason, on which we mainly concentrate here, is the necessity to check the so-called "bootstrap" conditions [14]. The matter is the following: the base of the BFKL equation approach is the property of the "gluon Reggeization", whose exact meaning was explained in details in Ref. [14]. This property was proved only in the LLA [15], while beyond the LLA it was checked only in the first three orders of the perturbation theory. The "bootstrap" conditions, obtained in Ref. [14], are just appealed to demonstrate the self-consistency of the BFKL approach in the NLA, although (if satisfied) they cannot be considered a proof of the Reggeization in a mathematical sense. The fulfillment of these conditions, however, would confirm so strongly the Reggeization that there would be no doubts that it is correct. Moreover, the check of the "bootstrap" equations is extremely important since they involve almost all the values appearing in the NLA BFKL kernel, so that the check provides a global test of the calculations [3]-[8] of the NLA corrections, which only in a small part were independently performed [8] or checked [16, 17].

The first bootstrap equation derived in [14] connects the kernel of the non-forward BFKL equation for the color octet in the t -channel with the gluon trajectory. In Ref. [18] it was shown that this equation is satisfied in the part concerning the quark contribution for arbitrary space-time dimension. The second bootstrap condition involves the impact factors of the scattered particles with color octet in the t -channel. The case of colliding

gluons is the object of the present paper, while quarks have been considered in a related paper [19].

The paper is organized as follows. In the next Section we explain the method of calculation, Sections 3, 4 and 5 are devoted to the calculation of one-gluon, quark-antiquark and two-gluon contributions to the gluon impact factors, respectively, for arbitrary color group representation in the t -channel. Section 6 contains details of the check of the second bootstrap condition, which involves the octet gluon impact factors in the NLA. The NLA gluon impact factors for the case of QCD with massless quark flavors are considered in the Section 7. The results obtained are briefly discussed in Section 8. Some integrations are carried out in the Appendix A.

2 Method of calculation

Let us remind that the impact factors were introduced in the BFKL approach for the description of the elastic scattering amplitudes $A + B \rightarrow A' + B'$ in the Regge kinematical region

$$s = (p_A + p_B)^2 = (p'_A + p'_B)^2 \rightarrow \infty, \quad t = (p_A - p'_A)^2 = (p'_B - p_B)^2 \quad \text{--fixed}, \quad (2.1)$$

where p_A , p_B and p'_A, p'_B are the momenta of the initial and final particles, respectively. We use for all vectors the Sudakov decomposition

$$p = \beta p_1 + \alpha p_2 + p_\perp, \quad p_\perp^2 = -\vec{p}^2, \quad (2.2)$$

the vectors (p_1, p_2) being the light-cone basis of the initial particle momenta plane (p_A, p_B) , so that we can put

$$p_A = p_1 + \frac{m_A^2}{s} p_2, \quad p_B = p_2 + \frac{m_B^2}{s} p_1. \quad (2.3)$$

Here m_A and m_B are the masses of the colliding particles A and B and the vector notation is used throughout this paper for the transverse components of the momenta, since all vectors in the transverse subspace are evidently space-like.

The basis of the BFKL approach is the gluon Reggeization. In the case of the elastic scattering, it means that the amplitude with gluon quantum numbers and negative signature in the t -channel has the Regge form

$$(\mathcal{A}_8^{(-)})_{AB}^{A'B'} = \Gamma_{A'A}^c \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{+s}{-t} \right)^{j(t)} \right] \Gamma_{B'B}^c. \quad (2.4)$$

Here c is a color index, $\Gamma_{P'P}^c$ are the particle-particle-Reggeon (PPR) vertices which do not depend on s and $j(t) = 1 + \omega(t)$ is the Reggeized gluon trajectory. In the derivation of the BFKL equation in the NLA it was assumed that this form, as well as the multi-Regge form of production amplitudes (see, for instance, [14] and references therein) is valid also in the NLA. Then, the s -channel unitarity of the scattering matrix leads to

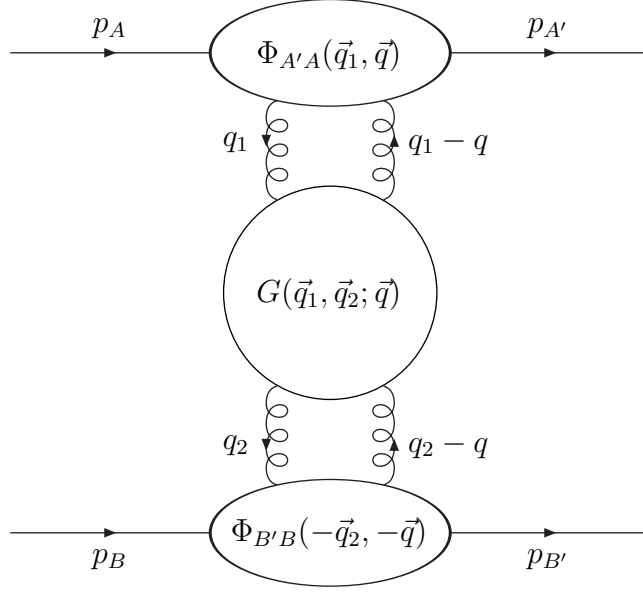


Figure 1: Diagrammatic representation of the elastic scattering amplitude $A + B \rightarrow A' + B'$.

$$\begin{aligned} \mathcal{I}m_s \left((\mathcal{A}_{AB})^{A'B'} \right) &= \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^2 \vec{q}_1'^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^2 \vec{q}_2'^2} \\ &\times \sum_{\nu} \Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1, \vec{q}; s_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) \right] \Phi_{B'B}^{(\mathcal{R},\nu)}(-\vec{q}_2, -\vec{q}; s_0) , \end{aligned} \quad (2.5)$$

where the momenta are defined in Fig. 1. For convenience we have introduced the notation (which will be used also in the following) $q'_i \equiv q_i - q$, where $q \simeq q_{\perp}$ is the momentum transfer in the process $A + B \rightarrow A' + B'$. We emphasize that the wavy intermediate lines in Fig. 1 denote Reggeons and not gluons – the Reggeons would only become gluons in absence of interaction. The space-time dimension, D , is taken to be $D = 4 + 2\epsilon$ in order to regularize the infrared divergences. In the above equation $\mathcal{A}_{\mathcal{R}}$ stands for the scattering amplitude with the irreducible representation \mathcal{R} of the color group in the t -channel, the index ν enumerates the states in this representation, $\Phi_{P'P}^{(\mathcal{R},\nu)}$ are the impact factors and $G_{\omega}^{(\mathcal{R})}$ is the Mellin transform of the Green function for the Reggeon-Reggeon scattering [14]. Here and below we do not indicate the signature, since it is defined by the symmetry of the representation \mathcal{R} in the product of the two octet representations. The parameter s_0 is an arbitrary energy scale introduced in order to define the partial wave expansion of the scattering amplitudes. The dependence on this parameter disappears in the full expressions for the amplitudes. The Green function obeys the generalized BFKL equation

$$\omega G_{\omega}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) = \vec{q}_1^2 \vec{q}_1'^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_r}{\vec{q}_r^2 \vec{q}_r'^2} \mathcal{K}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_r, \vec{q}) G_{\omega}^{(\mathcal{R})}(\vec{q}_r, \vec{q}_2, \vec{q}) , \quad (2.6)$$

where $\mathcal{K}^{(\mathcal{R})}$ is the kernel in the NLA [14].

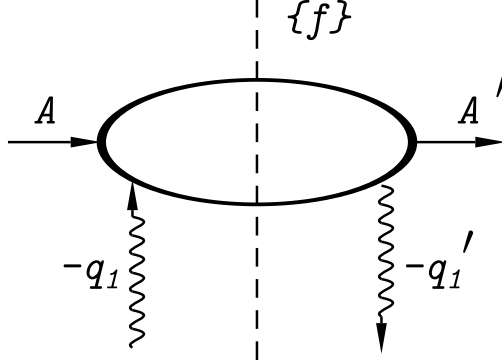


Figure 2: Schematic description of the intermediate states contributions to the impact factors.

The bootstrap conditions appear from the requirement that the imaginary part of the amplitude (2.4) must coincide with the R.H.S. of Eq. (2.5) in the case of gluon quantum numbers in the t -channel. The second bootstrap condition in the NLA reads [14]

$$- \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{q}_1^2 \vec{q}_1'^2} ig \sqrt{N} \Phi_{A'A}^{(8,a)(1)}(\vec{q}_1, \vec{q}; s_0) =$$

$$\Gamma_{A'A}^{(a)(1)} \omega^{(1)}(-\vec{q}^2) + \frac{1}{2} \Gamma_{A'A}^{(a)(B)} \left[\omega^{(2)}(-\vec{q}^2) + \left(\omega^{(1)}(-\vec{q}^2) \right)^2 \ln \left(\frac{s_0}{\vec{q}^2} \right) \right]. \quad (2.7)$$

Here g is the gauge coupling constant ($g^2 = 4\pi\alpha_s$), N is the number of colors, $\omega^{(1)}$ and $\omega^{(2)}$ are the one- and two-loop contributions to the Reggeized gluon trajectory, $\Gamma_{A'A}^{(a)(B)}$ and $\Gamma_{A'A}^{(a)(1)}$ are the Born and one-loop parts of the particle-particle-Reggeon (PPR) effective vertex. The definition of the non-forward impact factors with color state ν of the irreducible representation \mathcal{R} was given in Ref. [14] and can be presented as

$$\Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1, \vec{q}; s_0) = \langle cc' | \hat{\mathcal{P}}_{\mathcal{R}} | \nu \rangle \Phi_{AA'}^{cc'}(\vec{q}_1, \vec{q}; s_0), \quad (2.8)$$

where $\hat{\mathcal{P}}_{\mathcal{R}}$ is the projector of two-gluon color states in the t -channel on the irreducible representation \mathcal{R} . The value $\Phi_{AA'}^{cc'}$ determines completely the impact factors of the particle A with any possible color structure. We will consider mainly just this object and call it unprojected impact factor. The definition of such impact factors in the NLA can be reconstructed from the one of Ref. [14]:

$$\Phi_{AA'}^{cc'}(\vec{q}_1, \vec{q}; s_0) = \left(\frac{s_0}{\vec{q}_1^2} \right)^{\frac{1}{2}\omega(-\vec{q}_1^2)} \left(\frac{s_0}{\vec{q}_1'^2} \right)^{\frac{1}{2}\omega(-\vec{q}_1'^2)} \sum_{\{f\}} \int \theta(s_{\Lambda} - s_{AR}) \frac{ds_{AR} d\rho_f}{(2\pi)} \Gamma_{\{f\}A}^c$$

$$\times \left(\Gamma_{\{f\}A'}^{c'} \right)^* - \frac{1}{2} \int \frac{d^{D-2}q_2}{\vec{q}_2^2 \vec{q}_2'^2} \Phi_{AA'}^{c_1 c_1' (B)}(\vec{q}_2, \vec{q}) (\mathcal{K}_r^B)_{c_1 c}^{c_1' c'}(\vec{q}_2, \vec{q}_1, \vec{q}) \ln \left(\frac{s_{\Lambda}^2}{s_0 (\vec{q}_2 - \vec{q}_1)^2} \right). \quad (2.9)$$

In this expression it is enough to take the Reggeized gluon trajectory $\omega(t)$ in the one-loop approximation. For brevity, we do not perform here and below an explicit expansion in

α_s ; evidently, this expansion is assumed and only the leading and the next-to-leading terms should be kept. $\Gamma_{\{f\}A}^c$ is the effective vertex for production of the system $\{f\}$ (see Fig. 2) in the collision of the particle A and the Reggeized gluon with color index c and momentum

$$-q_1 = \alpha p_2 - q_{1\perp}, \quad \alpha \approx (s_{AR} - m_A^2 + \vec{q}_1^2)/s \ll 1, \quad (2.10)$$

and s_{AR} is the particle-Reggeon squared invariant mass. In the fragmentation region of the particles A and A' , where all transverse momenta as well as the invariant mass $\sqrt{s_{AR}}$ are not growing with s , we have for both Reggeons the relations

$$q_1^2 = -\vec{q}_1^2, \quad q_1'^2 = -\vec{q}_1'^2 = -(\vec{q}_1 - \vec{q})^2. \quad (2.11)$$

Summation in Eq. (2.9) is carried out over all systems $\{f\}$ which can be produced in the NLA and the integration is performed over the phase space volume of the produced system, which for a n -particle system (if there are identical particles in this system, corresponding factors should also be introduced) reads

$$d\rho_f = (2\pi)^D \delta^{(D)}\left(p_A - q_1 - \sum_{m=1}^n k_m\right) \prod_{m=1}^n \frac{d^{D-1}k_m}{2\epsilon_m (2\pi)^{D-1}}, \quad (2.12)$$

as well as over the particle-Reggeon invariant mass. The parameter s_Λ , limiting the integration region over the invariant mass in the first term in the R.H.S. of Eq. (2.9), is introduced for the separation of the contributions of multi-Regge and quasi-multi-Regge kinematics (MRK and QMRK) and should be considered as tending to infinity. The dependence of the impact factors on this parameter disappears [14] due to the cancellation between the first and the second term in the R.H.S. of Eq. (2.9). In the second term, $\Phi_{AA'}^{cc'(B)}$ is the Born contribution to the impact factor, which does not depend on s_0 (for this reason we omitted this argument there), while $(\mathcal{K}_r^B)_{c_1c'}^{c'_1c'}$ is the part of the unprojected non-forward BFKL kernel in the Born approximation connected with real particle production:

$$(\mathcal{K}_r^B)_{c_1c'}^{c'_1c'}(\vec{q}_1, \vec{q}_2, \vec{q}) = \frac{g^2}{(2\pi)^{D-1}} T_{c_1c}^d T_{c'_1c'}^d \left(\frac{\vec{q}_1^2 \vec{q}_2'^2 + \vec{q}_2^2 \vec{q}_1'^2}{(\vec{q}_1 - \vec{q}_2)^2} - \vec{q}^2 \right), \quad (2.13)$$

being T the color group generator in the adjoint representation. Let us note that the definitions in Eqs. (2.8) and (2.9) imply a suitable normalization of the amplitudes $\Gamma_{\{f\}A}^c$, namely that applied in Eq. (11) of Ref. [14] (see also there the text after Eq. (27)). We should also note, that the definitions (2.8) and (2.9) are applicable for the case of colorless particles as well as for the case of charged QCD particles, while the octet impact factors $\Phi_{A'A}^{(8,a)}$, entering the bootstrap condition (2.7), have sense, of course, only for colored particles.

Considering the impact factors of the particle A , we can without loss of generality assume the particle B to be massless, because the impact factors $\Phi_{AA'}^{cc'}$ are properties of the particle A only and cannot depend on the properties of the other scattered particle. So, everywhere below the initial particle momenta p_A and p_B are taken as the light-cone basis. For any gluon polarization vector we will use the light-cone gauge

$$e(k) p_B = 0 \quad (2.14)$$

and from the transversality of this vector to the gluon momentum k it is easy to get the following Sudakov representation:

$$e(k) = -\frac{(k_\perp e_\perp(k))}{(kp_B)} p_B + e_\perp(k) . \quad (2.15)$$

The transverse polarization vectors have the properties

$$(e_\perp^*(k, \lambda_1) e_\perp(k, \lambda_2)) = (e^*(k, \lambda_1) e(k, \lambda_2)) = -\delta_{\lambda_1, \lambda_2},$$

$$\sum_\lambda e_\perp^{*\mu}(k, \lambda) e_\perp^\nu(k, \lambda) = -g_{\perp\perp}^{\mu\nu} , \quad (2.16)$$

where the index λ enumerates the independent polarizations of gluon, $g^{\mu\nu}$ is the metrical tensor in the full space and $g_{\perp\perp}^{\mu\nu}$ the one in the transverse subspace,

$$g_{\perp\perp}^{\mu\nu} = g^{\mu\nu} - \frac{p_A^\mu p_B^\nu + p_B^\mu p_A^\nu}{(p_A p_B)} . \quad (2.17)$$

With the NLA accuracy the intermediate states $\{f\}$, which can contribute to the impact factors (2.9) in the gluon case, are one-gluon, two-gluon and quark-antiquark-pair states. In the case of the two-gluon contribution, we include the second term in the R.H.S. of Eq. (2.9), which is a counterterm for the LLA part contained in the first term. In the Born and quark-antiquark contributions to the gluon impact factors we will omit the argument s_0 , because of their evident independence on it. Firstly, we will consider the general case of arbitrary $\epsilon = (D - 4)/2$ and arbitrary mass m_f for the quark flavor f . Under these general conditions, we will determine integral representations for the gluon impact factors. These general integral representations are necessary for the check of the bootstrap condition (2.7). Then we will perform the integration in the expansion in ϵ for the practically important case of QCD with n_f massless quark flavors.

3 One-gluon contribution

In the case of the one-gluon contribution, the invariant mass $\sqrt{s_{AR}}$ is fixed to be zero, because of the masslessness of the intermediate gluon G , and one easily gets from the definition (2.9)

$$\Phi_{AA'}^{cc'\{G\}}(\vec{q}_1, \vec{q}; s_0) = \left(\frac{s_0}{\vec{q}_1^2}\right)^{\frac{1}{2}\omega(-\vec{q}_1^2)} \left(\frac{s_0}{\vec{q}'_1^2}\right)^{\frac{1}{2}\omega(-\vec{q}'_1^2)} \sum_\lambda \Gamma_{GA}^c (\Gamma_{GA'}^{c'})^* , \quad (3.1)$$

where the gluon-gluon-Reggeon (GGR) effective vertex Γ_{GA}^c , obtained in Refs. [6, 20] and [21], has the form

$$\Gamma_{GA}^c = gT_{GA}^c \left[\delta_{\lambda, \lambda_A} \left(1 + \Gamma_{GG}^{(+)(1)}(q_1^2) \right) + \delta_{\lambda, -\lambda_A} \Gamma_{GG}^{(-)(1)}(q_1^2) \right] . \quad (3.2)$$

Here $\Gamma_{GG}^{(\pm)(1)}$ represent the radiative corrections to the helicity conserving and non-conserving parts of the vertex, the δ 's on the helicities λ_A and λ of the gluons A and G , respectively, are determined by their form in the tensor representation

$$\delta_{\lambda, \pm\lambda_A} = e_\mu^* e_{A\mu} \delta_{\lambda, \pm\lambda_A}^{\mu\mu_A}, \quad \delta_{\lambda, \lambda_A}^{\mu\mu_A} = - \left(g^{\mu_A \rho} - \frac{p_A^\mu p_B^\rho + p_B^\mu p_A^\rho}{(p_A p_B)} \right) \left(g^{\mu\nu} - \frac{p^\mu p_B^\nu + p_B^\mu p^\nu}{(p p_B)} \right) g_{\rho\nu} ,$$

$$\delta_{\lambda, -\lambda_A}^{\mu\mu_A} = - \left(g^{\mu_A \rho} - \frac{p_A^{\mu_A} p_B^{\rho} + p_B^{\mu_A} p_A^{\rho}}{(p_A p_B)} \right) \left(g^{\mu \nu} - \frac{p^{\mu} p_B^{\nu} + p_B^{\mu} p^{\nu}}{(p p_B)} \right) \left(g_{\rho \nu} - (D-2) \frac{q_{1\rho} q_{1\nu}}{q_1^2} \right), \quad (3.3)$$

where p is the momentum of the intermediate gluon G . Using the Eqs. (2.15) and (3.3), it is easy to obtain the following relations:

$$\begin{aligned} \delta_{\lambda, \lambda_A} &= -e_{\perp}^{*\mu} e_{A\perp}^{\nu} g_{\mu\nu}^{\perp\perp}, & \delta_{\lambda, -\lambda_A} &= -e_{\perp}^{*\mu} e_{A\perp}^{\nu} T_{\mu\nu}^{\perp\perp}(q_{1\perp}), \\ \delta_{\lambda, \lambda_{A'}} &= -e_{\perp}^{*\mu} e_{A'\perp}^{\nu} g_{\mu\nu}^{\perp\perp}, & \delta_{\lambda, -\lambda_{A'}} &= -e_{\perp}^{*\mu} e_{A'\perp}^{\nu} T_{\mu\nu}^{\perp\perp}(q'_{1\perp}), \end{aligned} \quad (3.4)$$

with

$$T_{\mu\nu}^{\perp\perp}(k_{\perp}) = g_{\mu\nu}^{\perp\perp} - (D-2) \frac{k_{\mu}^{\perp} k_{\nu}^{\perp}}{k_{\perp}^2}. \quad (3.5)$$

From these relations one can get without any difficulty

$$\begin{aligned} \sum_{\lambda} \delta_{\lambda, \lambda_A} \delta_{\lambda, \lambda_{A'}}^* &= -e_{A'\perp}^{*\mu} e_{A\perp}^{\nu} g_{\mu\nu}^{\perp\perp}, \\ \sum_{\lambda} \delta_{\lambda, -\lambda_A} \delta_{\lambda, \lambda_{A'}}^* &= -e_{A'\perp}^{*\mu} e_{A\perp}^{\nu} T_{\mu\nu}^{\perp\perp}(q_{1\perp}), \\ \sum_{\lambda} \delta_{\lambda, \lambda_A} \delta_{\lambda, -\lambda_{A'}}^* &= -e_{A'\perp}^{*\mu} e_{A\perp}^{\nu} T_{\mu\nu}^{\perp\perp}(q'_{1\perp}), \end{aligned} \quad (3.6)$$

which gives for the convolution in Eq. (3.1):

$$\begin{aligned} \Phi_{AA'}^{cc'\{G\}}(\vec{q}_1, \vec{q}; s_0) &= -g^2 (T^{c'} T^c)_{A'A} e_{A'\perp}^{*\mu} e_{A\perp}^{\nu} \left[g_{\mu\nu}^{\perp\perp} \left(1 + \Gamma_{GG}^{(+)(1)}(-\vec{q}_1^2) \right. \right. \\ &\quad \left. \left. + \Gamma_{GG}^{(+)(1)}(-\vec{q}'^2) + \frac{1}{2} \omega^{(1)}(-\vec{q}_1^2) \ln \left(\frac{s_0}{\vec{q}_1^2} \right) + \frac{1}{2} \omega^{(1)}(-\vec{q}'^2) \ln \left(\frac{s_0}{\vec{q}_1'^2} \right) \right) \right. \\ &\quad \left. + T_{\mu\nu}^{\perp\perp}(q_{1\perp}) \Gamma_{GG}^{(-)(1)}(-\vec{q}_1^2) + T_{\mu\nu}^{\perp\perp}(q'_{1\perp}) \Gamma_{GG}^{(-)(1)}(-\vec{q}'^2) \right]. \end{aligned} \quad (3.7)$$

We need now the expressions for the radiative corrections $\Gamma_{GG}^{(\pm)(1)}$, which can be found in Ref. [20]. We present these expressions in a form slightly different from the one used there:

$$\begin{aligned} \Gamma_{GG}^{(+)(1)}(-\vec{v}^2) &= g^2 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \frac{1}{\epsilon} \left(-\frac{2}{\epsilon} + \frac{9(1+\epsilon)^2 + 2}{2(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} \right. \\ &\quad \left. + 2\psi(1+\epsilon) - \psi(1-\epsilon) - \psi(1) \right) (\vec{v}^2)^{\epsilon} + g^2 \frac{\Gamma(1-\epsilon)}{(1+\epsilon)(4\pi)^{2+\epsilon}} \\ &\quad \times \left[\sum_f \frac{2(1+\epsilon)}{\epsilon} \int_0^1 dx x(1-x) \left(m_f^2 + x(1-x)\vec{v}^2 \right)^{\epsilon} - \frac{1}{2} F_1^{(+)}(\vec{v}^2) \right], \\ \Gamma_{GG}^{(-)(1)}(-\vec{v}^2) &= g^2 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \\ &\quad \times \frac{1}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} (\vec{v}^2)^{\epsilon} - g^2 \frac{\Gamma(1-\epsilon)}{(1+\epsilon)(4\pi)^{2+\epsilon}} 2F_1^{(-)}(\vec{v}^2), \end{aligned} \quad (3.8)$$

with

$$\begin{aligned}
F_1^{(+)}(\vec{v}^2) &= \sum_f \left[\frac{2}{3} (m_f^2)^\epsilon + \int_0^1 \int_0^1 dx_1 dx_2 \theta(1-x_1-x_2) (m_f^2 + x_1 x_2 \vec{v}^2)^\epsilon \right. \\
&\quad \times \left. \frac{\vec{v}^2 (x_1 + x_2) (1 + \epsilon - 2(x_1 + x_2)(1 - x_1 - x_2)) - 4m_f^2 (1 - x_1 - x_2)}{(m_f^2 + x_1 x_2 \vec{v}^2)} \right], \\
F_1^{(-)}(\vec{v}^2) &= \sum_f \int_0^1 \int_0^1 dx_1 dx_2 \theta(1-x_1-x_2) (m_f^2 + x_1 x_2 \vec{v}^2)^\epsilon \frac{\vec{v}^2 x_1 x_2 (1 - x_1 - x_2)}{(m_f^2 + x_1 x_2 \vec{v}^2)}. \quad (3.9)
\end{aligned}$$

In these equations $\Gamma(x)$ and $\psi(x)$ are the Euler gamma-function and its logarithmic derivative, respectively. In order to pass from the representation of Ref. [20] for the radiative corrections $\Gamma_{GG}^{(\pm)(1)}$ to our expressions, we have used the identity

$$\begin{aligned}
0 &= \frac{1}{2} F_1^{(+)}(\vec{v}^2) + \sum_f \left[\frac{2(1+\epsilon)}{3\epsilon} (m_f^2)^\epsilon - \int_0^1 \int_0^1 dx_1 dx_2 \theta(1-x_1-x_2) (m_f^2 + x_1 x_2 \vec{v}^2)^\epsilon \right. \\
&\quad \times \left. \frac{1}{(m_f^2 + x_1 x_2 \vec{v}^2)} \left(\frac{(1+\epsilon)}{\epsilon} (2 - x_1 - x_2) (m_f^2 + (1+2\epsilon)\vec{v}^2 x_1 x_2) - 2\vec{v}^2 x_1 x_2 (1 - x_1 - x_2) \right) \right], \quad (3.10)
\end{aligned}$$

that can be found also in Ref. [20]. We need also the expression for the one-loop Reggeized gluon trajectory $\omega^{(1)}$ [1]

$$\omega^{(1)}(-\vec{v}^2) = -\frac{g^2 N}{2} \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \frac{\vec{v}^2}{\vec{k}^2 (\vec{k} - \vec{v})^2} = -g^2 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \frac{2}{\epsilon} (\vec{v}^2)^\epsilon. \quad (3.11)$$

Using Eqs. (3.7)-(3.11), we obtain finally the Born impact factor,

$$\Phi_{AA'}^{cc'(B)}(\vec{q}_1, \vec{q}) = -g^2 (T^{c'} T^c)_{A'A} e_{A'\perp}^{*\mu} e_{A\perp}^\nu g_{\mu\nu}^{\perp\perp}, \quad (3.12)$$

and the NLA correction to the impact factors due to the one-gluon contribution,

$$\begin{aligned}
\Phi_{AA'}^{cc'(1)\{G\}}(\vec{q}_1, \vec{q}; s_0) &= -N (T^{c'} T^c)_{A'A} g^4 \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} e_{A'\perp}^{*\mu} e_{A\perp}^\nu \left[\left\{ g_{\mu\nu}^{\perp\perp} \frac{1}{\epsilon} \right. \right. \\
&\quad \times \left(-\ln\left(\frac{s_0}{\vec{v}^2}\right) - \frac{2}{\epsilon} + \frac{9(1+\epsilon)^2 + 2}{2(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} + 2\psi(1+\epsilon) - \psi(1-\epsilon) - \psi(1) \right) (\vec{v}^2)^\epsilon \\
&\quad \left. + \frac{1}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} (\vec{v}^2)^\epsilon T_{\mu\nu}^{\perp\perp}(v_\perp) \right\} \Big|_{v_\perp=q'_{1\perp}} + \left\{ \dots \right\} \Big|_{v_\perp=q_{1\perp}} \Big] - (T^{c'} T^c)_{A'A} \\
&\quad \times g^4 \frac{\Gamma(1-\epsilon)}{(1+\epsilon)(4\pi)^{2+\epsilon}} e_{A'\perp}^{*\mu} e_{A\perp}^\nu \left[\left\{ g_{\mu\nu}^{\perp\perp} \left(\sum_f \frac{2(1+\epsilon)}{\epsilon} \int_0^1 dx x (1-x) (m_f^2 + x(1-x)\vec{v}^2)^\epsilon \right. \right. \right. \\
&\quad \left. \left. \left. - \frac{1}{2} F_1^{(+)}(\vec{v}^2) \right) - 2T_{\mu\nu}^{\perp\perp}(v_\perp) F_1^{(-)}(\vec{v}^2) \right\} \Big|_{v_\perp=q'_{1\perp}} + \left\{ \dots \right\} \Big|_{v_\perp=q_{1\perp}} \right]. \quad (3.13)
\end{aligned}$$

4 Quark-antiquark-pair contribution

In this section we calculate the pure NLA contribution to the gluon impact factors defined in Eq. (2.9), coming from a quark-antiquark-pair production in the fragmentation region:

$$\Phi_{AA'}^{cc'(1)\{q\bar{q}\}}(\vec{q}_1, \vec{q}) = \sum_f \sum_{\substack{\lambda_1, \lambda_2 \\ i_1, i_2}} \int \frac{ds_{AR} d\rho_{\{q\bar{q}\}}}{(2\pi)} \Gamma_{\{q\bar{q}\}A}^c \left(\Gamma_{\{q\bar{q}\}A'}^{c'} \right)^* , \quad (4.1)$$

where the summation is performed over the quark flavors f , over the helicities λ_1 and λ_2 and over the color indices i_1 and i_2 of the produced quark and antiquark with momenta k_1 and k_2 , respectively. Integration over the invariant mass here is convergent and we do not need to introduce the cutoff s_Λ . As usual, we use the Sudakov representation

$$k_{1,2} = \beta_{1,2} p_A + \frac{m_f^2 + \vec{k}_{1,2}^2}{s\beta_{1,2}} p_B + k_{1,2\perp} , \quad k_{1,2}^2 = m_f^2 , \quad (4.2)$$

where m_f is the quark mass. Then we have the relations

$$s_{AR} = (k_1 + k_2)^2 = \frac{m_f^2 + (\vec{k}_1\beta_2 - \vec{k}_2\beta_1)^2}{\beta_1\beta_2} , \quad (4.3)$$

$$\frac{ds_{AR} d\rho_{\{q\bar{q}\}}}{(2\pi)} = \delta(1 - \beta_1 - \beta_2) \delta^{(D-2)}((k_1 + k_2 + q_1)_\perp) \frac{d\beta_1 d\beta_2}{\beta_1\beta_2} \frac{d^{D-2}k_1 d^{D-2}k_2}{2(2\pi)^{D-1}} . \quad (4.4)$$

To obtain the last of these relations we have used also Eqs. (2.10) and (2.12). The amplitude of $q\bar{q}$ pair production in the gluon-Reggeon collision $\Gamma_{\{q\bar{q}\}A}^c$ was obtained in Ref. [20], but we will use a more convenient, slightly different form for it, which can be easily obtained from the expression presented there:

$$\begin{aligned} \Gamma_{\{q\bar{q}\}A}^c &= g^2 \left[\left(t^A t^c \right)_{i_1 i_2} \mathcal{A}_{q\bar{q}} - \left(t^c t^A \right)_{i_1 i_2} \mathcal{A}_{q\bar{q}}(1 \leftrightarrow 2) \right] , \\ \Gamma_{\{q\bar{q}\}A'}^{c'} &= g^2 \left[\left(t^{A'} t^{c'} \right)_{i_1 i_2} \mathcal{A}'_{q\bar{q}} - \left(t^{c'} t^{A'} \right)_{i_1 i_2} \mathcal{A}'_{q\bar{q}}(1 \leftrightarrow 2) \right] , \end{aligned} \quad (4.5)$$

where t^a are the color group generators in the fundamental representation. The amplitude $\mathcal{A}_{q\bar{q}}$ will be defined below.

Using the symmetry ($1 \leftrightarrow 2$) of the phase space volume (4.4) and the expressions (4.5), we get for the $q\bar{q}$ contribution (4.1) to the impact factors

$$\begin{aligned} \Phi_{AA'}^{cc'(1)\{q\bar{q}\}}(\vec{q}_1, \vec{q}) &= g^4 \sum_f \int_0^1 \int \frac{d\beta}{\beta(1-\beta)} \frac{d^{D-2}k_1}{2(2\pi)^{D-1}} \\ &\times \left[C_{1AA'}^{cc'} \sum_{\lambda_1, \lambda_2} \mathcal{A}_{q\bar{q}} \mathcal{A}_{q\bar{q}}'^* - C_{2AA'}^{cc'} \sum_{\lambda_1, \lambda_2} \mathcal{A}_{q\bar{q}} \mathcal{A}_{q\bar{q}}'^*(1 \leftrightarrow 2) \right] , \end{aligned} \quad (4.6)$$

where $\beta \equiv \beta_1$ and

$$C_{1AA'}^{cc'} = \text{tr} \left(t^A t^c t^{c'} t^{A'} + t^A t^{A'} t^{c'} t^c \right) , \quad C_{2AA'}^{cc'} = \text{tr} \left(t^A t^c t^{A'} t^{c'} + t^A t^{c'} t^A t^c \right) . \quad (4.7)$$

The amplitudes $\mathcal{A}_{q\bar{q}}$ and $\mathcal{A}'_{q\bar{q}}$ have the forms

$$\begin{aligned}\mathcal{A}_{q\bar{q}} &= \bar{u}_1 \frac{\not{p}_B}{s} \left(\not{\epsilon}_{A\perp} \not{S}_{12\perp} - 2(1-\beta) (e_{A\perp} S_{12\perp}) - \not{\epsilon}_{A\perp} m_f S_{12} \right) v_2, \\ \mathcal{A}'_{q\bar{q}} &= \bar{u}_1 \frac{\not{p}_B}{s} \left(\not{\epsilon}_{A'\perp} \not{S}'_{12\perp} - 2(1-\beta) (e_{A'\perp} S'_{12\perp}) - \not{\epsilon}_{A'\perp} m_f S'_{12} \right) v_2,\end{aligned}\quad (4.8)$$

with

$$\begin{aligned}S_{12\perp} &= \left(\frac{k_1}{k_{1\perp}^2 - m_f^2} - \frac{(k_1 + \beta q_1)}{(k_1 + \beta q_1)_\perp^2 - m_f^2} \right)_\perp, \\ S'_{12\perp} &= \left(\frac{(k_1 + \beta q)}{(k_1 + \beta q)_\perp^2 - m_f^2} - \frac{(k_1 + \beta q_1)}{(k_1 + \beta q_1)_\perp^2 - m_f^2} \right)_\perp, \\ S_{12} &= \frac{1}{k_{1\perp}^2 - m_f^2} - \frac{1}{(k_1 + \beta q_1)_\perp^2 - m_f^2}, \\ S'_{12} &= \frac{1}{(k_1 + \beta q)_\perp^2 - m_f^2} - \frac{1}{(k_1 + \beta q_1)_\perp^2 - m_f^2}.\end{aligned}\quad (4.9)$$

In these relations u_1 and v_2 are the spin wave functions of the final quark and antiquark, respectively. The other amplitude which enters the expression (4.6) for the $q\bar{q}$ contribution to the impact factors is

$$\mathcal{A}'_{q\bar{q}}(1 \leftrightarrow 2) = -\bar{u}_1 \frac{\not{p}_B}{s} \left(\not{\epsilon}_{A'\perp} \not{S}'_{21\perp} - 2(1-\beta) (e_{A'\perp} S'_{21\perp}) + \not{\epsilon}_{A'\perp} m_f S'_{21} \right) v_2, \quad (4.10)$$

with

$$\begin{aligned}S'_{21\perp} &= S'_{12\perp}(k_1 \leftrightarrow k_2) = \left(-\frac{(k_1 + q'_1 + \beta q)}{(k_1 + q'_1 + \beta q)_\perp^2 - m_f^2} + \frac{(k_1 + \beta q_1)}{(k_1 + \beta q_1)_\perp^2 - m_f^2} \right)_\perp, \\ S'_{21} &= S'_{12}(k_1 \leftrightarrow k_2) = \frac{1}{(k_1 + q'_1 + \beta q)_\perp^2 - m_f^2} - \frac{1}{(k_1 + \beta q_1)_\perp^2 - m_f^2}.\end{aligned}\quad (4.11)$$

The amplitude $\mathcal{A}'_{q\bar{q}}(1 \leftrightarrow 2)$ is obtained from the amplitude $\mathcal{A}'_{q\bar{q}}$ by the replacement $(1 \leftrightarrow 2)$, as it can be seen using the charge conjugation. An analogous expression can be written also for the amplitude $\mathcal{A}_{q\bar{q}}(1 \leftrightarrow 2)$, but we do not give it here, because it does not enter Eq. (4.6). Then convolutions in Eq. (4.6) are calculated immediately and give

$$\begin{aligned}\sum_{\lambda_1, \lambda_2} \mathcal{A}_{q\bar{q}} \mathcal{A}_{q\bar{q}}'^* &= 2\beta(1-\beta) e_{A'\mu}^* e_{A\nu}^\perp \left[g_{\perp\perp}^{\mu\nu} \left((S'_{12\perp} S_{12\perp}) \right. \right. \\ &\quad \left. \left. - m_f^2 S'_{12} S_{12} \right) - S_{12\perp}'^\nu S_{12\perp}^\mu + (1 - 4\beta(1-\beta)) S_{12\perp}'^\mu S_{12\perp}^\nu \right], \\ - \sum_{\lambda_1, \lambda_2} \mathcal{A}_{q\bar{q}} \mathcal{A}_{q\bar{q}}'^*(1 \leftrightarrow 2) &= 2\beta(1-\beta) e_{A'\mu}^* e_{A\nu}^\perp \left[g_{\perp\perp}^{\mu\nu} \left((S'_{21\perp} S_{12\perp}) \right. \right. \\ &\quad \left. \left. + m_f^2 S'_{21} S_{12} \right) - S_{21\perp}'^\nu S_{12\perp}^\mu + (1 - 4\beta(1-\beta)) S_{21\perp}'^\mu S_{12\perp}^\nu \right].\end{aligned}\quad (4.12)$$

This result allows us to present the quark-antiquark contribution (4.1) to the impact factors in the following form:

$$\Phi_{AA'}^{cc'(1)\{q\bar{q}\}}(\vec{q}_1, \vec{q}) = C_{1AA'}^{cc'} I_1 + C_{2AA'}^{cc'} I_2, \quad (4.13)$$

with

$$\begin{aligned} I_1 = & g^4 e_{A'\mu}^{*\perp} e_{A\nu}^\perp \sum_f \left[g_{\perp\perp}^{\mu\nu} \left(\int_0^1 d\beta \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} (S'_{12\perp} S_{12\perp}) - m_f^2 \int_0^1 d\beta \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S'_{12} S_{12} \right) \right. \\ & \left. - \int_0^1 d\beta \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S'_{12\perp}^\nu S_{12\perp}^\mu + \int_0^1 d\beta (1 - 4\beta(1 - \beta)) \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S'_{12\perp}^\mu S_{12\perp}^\nu \right] \end{aligned} \quad (4.14)$$

and

$$\begin{aligned} I_2 = & g^4 e_{A'\mu}^{*\perp} e_{A\nu}^\perp \sum_f \left[g_{\perp\perp}^{\mu\nu} \left(\int_0^1 d\beta \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} (S'_{21\perp} S_{12\perp}) + m_f^2 \int_0^1 d\beta \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S'_{21} S_{12} \right) \right. \\ & \left. - \int_0^1 d\beta \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S'_{21\perp}^\nu S_{12\perp}^\mu + \int_0^1 d\beta (1 - 4\beta(1 - \beta)) \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S'_{21\perp}^\mu S_{12\perp}^\nu \right]. \end{aligned} \quad (4.15)$$

Let us consider first I_1 . Using the relations

$$\begin{aligned} \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S'_{12\perp}^\nu S_{12\perp}^\mu &= \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S'_{12\perp}^\mu S_{12\perp}^\nu = \beta^{2\epsilon} \left[J_1^{\mu\nu} \left(q_\perp, \frac{m_f^2}{\beta^2} \right) \right. \\ & \left. + J_1^{\mu\nu} \left(0_\perp, \frac{m_f^2}{\beta^2} \right) - J_1^{\mu\nu} \left(q'_{1\perp}, \frac{m_f^2}{\beta^2} \right) - J_1^{\mu\nu} \left(q_{1\perp}, \frac{m_f^2}{\beta^2} \right) \right], \end{aligned} \quad (4.16)$$

$$\begin{aligned} J_1^{\mu\nu} \left(v_\perp, \frac{m_f^2}{\beta^2} \right) &= \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \frac{k_\perp^\mu (k - v)_\perp^\nu}{\left(k_\perp^2 - \frac{m_f^2}{\beta^2} \right) \left((k - v)_\perp^2 - \frac{m_f^2}{\beta^2} \right)} = \\ \beta^{-2\epsilon} \frac{\Gamma(1 - \epsilon)}{(1 + \epsilon)(4\pi)^{2+\epsilon}} &\left[g_{\perp\perp}^{\mu\nu} \int_0^1 dx \left(m_f^2 + \beta^2 x(1 - x) \vec{v}^2 \right)^\epsilon \left(\frac{1 + 2\epsilon}{\epsilon} - \frac{m_f^2}{(m_f^2 + \beta^2 x(1 - x) \vec{v}^2)} \right) \right. \\ & \left. - T_{\perp\perp}^{\mu\nu}(v_\perp) \int_0^1 dx \left(m_f^2 + \beta^2 x(1 - x) \vec{v}^2 \right)^\epsilon \left(1 - \frac{m_f^2}{(m_f^2 + \beta^2 x(1 - x) \vec{v}^2)} \right) \right], \end{aligned} \quad (4.17)$$

$$\begin{aligned} \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S'_{12} S_{12} &= \beta^{2\epsilon-2} \left[J_1 \left(\vec{q}^2, \frac{m_f^2}{\beta^2} \right) + J_1 \left(0, \frac{m_f^2}{\beta^2} \right) \right. \\ & \left. - J_1 \left(\vec{q}'^2, \frac{m_f^2}{\beta^2} \right) - J_1 \left(\vec{q}_1^2, \frac{m_f^2}{\beta^2} \right) \right], \end{aligned} \quad (4.18)$$

$$J_1 \left(\vec{v}^2, \frac{m_f^2}{\beta^2} \right) = \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \frac{1}{\left(\vec{k}^2 + \frac{m_f^2}{\beta^2} \right) \left((\vec{k} - \vec{v})^2 + \frac{m_f^2}{\beta^2} \right)} =$$

$$2\beta^{2-2\epsilon} \frac{\Gamma(1 - \epsilon)}{(4\pi)^{2+\epsilon}} \int_0^1 dx \left(m_f^2 + \beta^2 x(1 - x) \vec{v}^2 \right)^\epsilon \frac{1}{(m_f^2 + \beta^2 x(1 - x) \vec{v}^2)}, \quad (4.19)$$

together with Eqs. (3.5), (3.9) and (4.14), performing also the following change of the integration variables in Eqs. (4.14) and (4.16)-(4.19):

$$x_1 = \beta x, \quad x_2 = \beta(1-x), \quad (4.20)$$

we can set I_1 in the form

$$I_1 = -g^4 \frac{\Gamma(1-\epsilon)}{(1+\epsilon)(4\pi)^{2+\epsilon}} e_{A'\perp}^{*\mu} e_{A\perp}^\nu \left[g_{\mu\nu}^{\perp\perp} \left(F_1^{(+)}(\vec{q}_1'^2) + F_1^{(+)}(\vec{q}_1^2) - F_1^{(+)}(\vec{q}^2) \right) \right. \\ \left. + 4T_{\mu\nu}^{\perp\perp}(q_{1\perp}') F_1^{(-)}(\vec{q}_1'^2) + 4T_{\mu\nu}^{\perp\perp}(q_{1\perp}) F_1^{(-)}(\vec{q}_1^2) - 4T_{\mu\nu}^{\perp\perp}(q_\perp) F_1^{(-)}(\vec{q}^2) \right]. \quad (4.21)$$

To calculate I_2 we use the following relations which can be obtained starting from Eqs. (4.9) and (4.11):

$$\int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S_{21\perp}'^\mu S_{12\perp}^\nu = \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} S_{21\perp}'^\nu S_{12\perp}^\mu = \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \left\{ \left[g_{\perp\perp}^{\mu\nu} \left(-\frac{1}{\epsilon} (m_f^2)^\epsilon \right. \right. \right. \\ \left. \left. + \frac{1}{\epsilon} \int_0^1 dx (m_f^2 + x(1-x)\vec{v}^2)^\epsilon \right) + 2 \int_0^1 dx (m_f^2 + x(1-x)\vec{v}^2)^\epsilon \right. \\ \left. \left. \times \frac{x(1-x)\vec{v}^2}{(m_f^2 + x(1-x)\vec{v}^2)} \frac{v_\perp^\mu v_\perp^\nu}{v_\perp^2} \right] \right|_{v_\perp=(1-\beta)q_{1\perp}'} + [\dots] \Big|_{v_\perp=\beta q_{1\perp}} - [\dots] \Big|_{v_\perp=((1-\beta)q_1'+\beta q_1)_\perp} \right\}, \quad (4.22)$$

$$\int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} m_f^2 S_{21}' S_{12} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \left\{ \left[(m_f^2)^\epsilon - \int_0^1 dx (m_f^2 + x(1-x)\vec{v}^2)^\epsilon \right. \right. \\ \left. \left. \times \frac{m_f^2}{(m_f^2 + x(1-x)\vec{v}^2)} \right] \right|_{\vec{v}=(1-\beta)\vec{q}_1'} + [\dots] \Big|_{\vec{v}=\beta\vec{q}_1} - [\dots] \Big|_{\vec{v}=(1-\beta)\vec{q}_1'+\beta\vec{q}_1} \right\}. \quad (4.23)$$

Then we get from Eq. (4.15)

$$I_2 = 2g^4 \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} e_{A'\perp}^{*\mu} e_{A\perp}^\nu \sum_f \left\{ \left[g_{\mu\nu}^{\perp\perp} \left(-\frac{2}{3\epsilon} (m_f^2)^\epsilon + \int_0^1 \int_0^1 d\beta dx (m_f^2 + x(1-x)\vec{v}^2)^\epsilon \right. \right. \right. \\ \left. \left. \times \left(\frac{1+2\epsilon-2\beta(1-\beta)}{\epsilon} - \frac{2m_f^2}{(m_f^2 + x(1-x)\vec{v}^2)} \right) \right) \right. \\ \left. - 4 \int_0^1 \int_0^1 d\beta dx \beta(1-\beta) (m_f^2 + x(1-x)\vec{v}^2)^\epsilon \frac{x(1-x)\vec{v}^2}{(m_f^2 + x(1-x)\vec{v}^2)} \right. \\ \left. \left. \times \frac{v_{\perp\mu} v_{\perp\nu}}{v_\perp^2} \right] \right|_{v_\perp=(1-\beta)q_{1\perp}'} + [\dots] \Big|_{v_\perp=\beta q_{1\perp}} - [\dots] \Big|_{v_\perp=((1-\beta)q_1'+\beta q_1)_\perp} \right\}. \quad (4.24)$$

Finally, the quark-antiquark contribution to the gluon impact factors is given by Eq. (4.13) with the color factors $C_{1,2AA'}^{cc'}$ defined in Eq. (4.7), I_1 and I_2 expressed in Eqs. (4.21) and (4.24), respectively, the functions $F_1^{(\pm)}$ being the same as in the previous section (see Eq. (3.9)).

5 Two-gluon contribution

The two-gluon contribution to the gluon impact factors defined in Eq. (2.9) can be presented as

$$\Phi_{AA'}^{cc'(1)\{GG\}}(\vec{q}_1, \vec{q}; s_0) = \sum_{\substack{\lambda_1, \lambda_2 \\ i_1, i_2}} \int \theta(s_\Lambda - s_{AR}) \frac{ds_{AR} d\rho_{\{GG\}}}{(2\pi)} \Gamma_{\{GG\}A}^c \left(\Gamma_{\{GG\}A'}^{c'} \right)^* , \quad (5.1)$$

where λ_1, λ_2 and i_1, i_2 are helicities and color indices of the produced gluons with momenta k_1 and k_2 . The expressions for the Sudakov representation of the produced gluon momenta and their invariant mass are the same as in the quark-antiquark case (see Eqs. (4.2) and (4.3)), but with $m_f = 0$. As for the integration volume element, we should introduce in Eq. (4.4) the factors $\theta(s_\Lambda - s_{AR})$ and $1/2!$ due to the gluons identity. The two-gluon production effective amplitude has the form [3]

$$\Gamma_{\{GG\}A}^c = 4g^2 q_1^2 \left[T_{i_1 A}^{c_1} T_{i_2 c}^{c_1} \mathcal{A}_{GG} + (1 \leftrightarrow 2) \right] , \quad (5.2)$$

while the other amplitude $\Gamma_{\{GG\}A'}^{c'}$ in Eq. (5.1) can be obtained from this relation by the evident substitutions $A \rightarrow A', c \rightarrow c'$. The gauge invariant expression for the amplitude \mathcal{A}_{GG} is rather complicated [3], but in our gauge (2.14) it becomes very simple [20] (see below). Just as in the quark-antiquark case, we can use the $(1 \leftrightarrow 2)$ symmetry of the integration volume element to obtain

$$\begin{aligned} \Phi_{AA'}^{cc'(1)\{GG\}}(\vec{q}_1, \vec{q}; s_0) &= 8g^4 q_1^2 q_1'^2 \int_0^1 \int \theta(s_\Lambda - s_{AR}) \frac{d\beta}{\beta(1-\beta)} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \\ &\times \left[C_{3AA'}^{cc'} \sum_{\lambda_1, \lambda_2} \mathcal{A}_{GG} \mathcal{A}_{GG}^* + C_{4AA'}^{cc'} \sum_{\lambda_1, \lambda_2} \mathcal{A}_{GG} \mathcal{A}_{GG}^* (1 \leftrightarrow 2) \right] , \end{aligned} \quad (5.3)$$

where $\beta \equiv \beta_1$ and

$$C_{3AA'}^{cc'} = (T^{c'_1} T^{c_1})_{A'A} (T^{c'_1} T^{c_1})_{c'c} , \quad C_{4AA'}^{cc'} = (T^{c'_1} T^{c_1})_{c'A} (T^{c'_1} T^{c_1})_{A'c} . \quad (5.4)$$

The amplitudes \mathcal{A}_{GG} , \mathcal{A}'_{GG} and $\mathcal{A}'_{GG}(1 \leftrightarrow 2)$ in the gauge (2.14, 2.15) have the forms

$$\begin{aligned} \mathcal{A}_{GG} &= \frac{1}{2q_1^2} \left[-\beta(1-\beta) (e_{1\perp}^* e_{2\perp}^*) (e_{A\perp} \tilde{R}_{12\perp}) \right. \\ &\quad \left. + \beta (e_{1\perp}^* e_{A\perp}) (e_{2\perp}^* \tilde{R}_{12\perp}) + (1-\beta) (e_{2\perp}^* e_{A\perp}) (e_{1\perp}^* \tilde{R}_{12\perp}) \right] , \\ \mathcal{A}'_{GG} &= \frac{1}{2q_1'^2} \left[-\beta(1-\beta) (e_{1\perp}^* e_{2\perp}^*) (e_{A'\perp} \tilde{R}'_{12\perp}) \right. \\ &\quad \left. + \beta (e_{1\perp}^* e_{A'\perp}) (e_{2\perp}^* \tilde{R}'_{12\perp}) + (1-\beta) (e_{2\perp}^* e_{A'\perp}) (e_{1\perp}^* \tilde{R}'_{12\perp}) \right] , \\ \mathcal{A}'_{GG}(1 \leftrightarrow 2) &= \frac{1}{2q_1'^2} \left[-\beta(1-\beta) (e_{1\perp}^* e_{2\perp}^*) (e_{A'\perp} \tilde{R}'_{21\perp}) \right. \end{aligned}$$

$$+ \beta (e_{1\perp}^* e_{A'\perp}) \left(e_{2\perp}^* \tilde{R}'_{21\perp} \right) + (1 - \beta) (e_{2\perp}^* e_{A'\perp}) \left(e_{1\perp}^* \tilde{R}'_{21\perp} \right) \Big] , \quad (5.5)$$

where e_1 and e_2 are the polarization vectors of the produced gluons and

$$\begin{aligned} \tilde{R}_{12\perp} &= \left(\frac{k_1}{k_{1\perp}^2} - \frac{(k_1 + \beta q_1)}{(k_1 + \beta q_1)_\perp^2} \right)_\perp , & \tilde{R}'_{12\perp} &= \left(\frac{(k_1 + \beta q)}{(k_1 + \beta q)_\perp^2} - \frac{(k_1 + \beta q_1)}{(k_1 + \beta q_1)_\perp^2} \right)_\perp , \\ \tilde{R}'_{21\perp} &= \tilde{R}_{12\perp} (1 \leftrightarrow 2) = \left(-\frac{(k_1 + q'_1 + \beta q)}{(k_1 + q'_1 + \beta q)_\perp^2} + \frac{(k_1 + \beta q_1)}{(k_1 + \beta q_1)_\perp^2} \right)_\perp . \end{aligned} \quad (5.6)$$

Eqs. (5.5) leads to the following expressions for the convolutions in the relation (5.3) for the two-gluon contribution to the impact factors:

$$\begin{aligned} \sum_{\lambda_1, \lambda_2} \mathcal{A}_{GG} \mathcal{A}_{GG}^* &= \frac{1}{4q_1^2 q_1'^2} \left[\left(\beta^2 + (1 - \beta)^2 \right) (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} \right. \\ &\quad \left. + 2\beta(1 - \beta) e_{A\mu}^\perp e_{A'\nu}^{*\perp} - 2\beta(1 - \beta) (1 - (1 + \epsilon)\beta(1 - \beta)) e_{A'\mu}^{*\perp} e_{A\nu}^\perp \right] \tilde{R}'_{12\perp}{}^\mu \tilde{R}_{12\perp}{}^\nu , \\ \sum_{\lambda_1, \lambda_2} \mathcal{A}_{GG} \mathcal{A}_{GG}^* (1 \leftrightarrow 2) &= \frac{1}{4q_1^2 q_1'^2} \left[\left(\beta^2 + (1 - \beta)^2 \right) (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} \right. \\ &\quad \left. + 2\beta(1 - \beta) e_{A\mu}^\perp e_{A'\nu}^{*\perp} - 2\beta(1 - \beta) (1 - (1 + \epsilon)\beta(1 - \beta)) e_{A'\mu}^{*\perp} e_{A\nu}^\perp \right] \tilde{R}'_{21\perp}{}^\mu \tilde{R}_{12\perp}{}^\nu , \end{aligned} \quad (5.7)$$

which allow us to get through Eqs. (5.3) the relation

$$\begin{aligned} \Phi_{AA'}^{cc'(1)\{GG\}}(\vec{q}_1, \vec{q}; s_0) &= 2g^4 \int_0^1 \int \theta(s_\Lambda - s_{AR}) \frac{d\beta}{\beta(1 - \beta)} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \left[\left(\beta^2 + (1 - \beta)^2 \right) \right. \\ &\quad \left. \times (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} + 2\beta(1 - \beta) e_{A\mu}^\perp e_{A'\nu}^{*\perp} - 2\beta(1 - \beta) (1 - (1 + \epsilon)\beta(1 - \beta)) e_{A'\mu}^{*\perp} e_{A\nu}^\perp \right] \\ &\quad \times \left[C_{3AA'}^{cc'} \tilde{R}'_{12\perp}{}^\mu \tilde{R}_{12\perp}{}^\nu + C_{4AA'}^{cc'} \tilde{R}'_{21\perp}{}^\mu \tilde{R}_{12\perp}{}^\nu \right] . \end{aligned} \quad (5.8)$$

It seems to be convenient now to change the integration momentum k_1 in the following way:

$$k_1 \rightarrow -\beta k_1 , \quad d^{D-2}k_1 \rightarrow \beta^{2+2\epsilon} d^{D-2}k_1 , \quad (5.9)$$

which states the substitutions

$$\begin{aligned} s_{AR} &\rightarrow \frac{\beta(\vec{k}_1 - \vec{q}_1)^2}{(1 - \beta)} , & \tilde{R}_{12\perp} &\rightarrow -\frac{1}{\beta} R_{12\perp} = -\frac{1}{\beta} \left(\frac{k_1}{k_{1\perp}^2} - \frac{(k_1 - q_1)}{(k_1 - q_1)_\perp^2} \right)_\perp , \\ \tilde{R}'_{12\perp} &\rightarrow -\frac{1}{\beta} R'_{12\perp} = -\frac{1}{\beta} \left(\frac{(k_1 - q)}{(k_1 - q)_\perp^2} - \frac{(k_1 - q_1)}{(k_1 - q_1)_\perp^2} \right)_\perp , \\ \tilde{R}'_{21\perp} &\rightarrow -\frac{1}{\beta} R'_{21\perp} = -\frac{1}{\beta} \left(-\frac{\beta(\beta k_1 - q'_1 - \beta q)}{(\beta k_1 - q'_1 - \beta q)_\perp^2} + \frac{(k_1 - q_1)}{(k_1 - q_1)_\perp^2} \right)_\perp . \end{aligned} \quad (5.10)$$

Consequently, the representation for the two-gluon part of the impact factors reads

$$\Phi_{AA'}^{cc'(1)\{GG\}}(\vec{q}_1, \vec{q}; s_0) = 2g^4 \int_0^1 \int \theta \left(1 - \frac{\beta(\vec{k}_1 - \vec{q}_1)^2}{(1 - \beta)s_\Lambda} \right) \frac{\beta^{2\epsilon-1} d\beta}{(1 - \beta)} \frac{d^{D-2}k_1}{(2\pi)^{D-1}}$$

$$\begin{aligned}
& \times \left[(\beta^2 + (1 - \beta)^2) (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} + 2\beta(1 - \beta) e_{A\mu}^\perp e_{A'\nu}^{*\perp} - 2\beta(1 - \beta) \right. \\
& \times (1 - (1 + \epsilon)\beta(1 - \beta)) e_{A'\mu}^{*\perp} e_{A\nu}^\perp \left. \right] \left[C_{3AA'}^{cc'} R_{12\perp}'^\mu R_{12\perp}^\nu + C_{4AA'}^{cc'} R_{21\perp}'^\mu R_{12\perp}^\nu \right]. \quad (5.11)
\end{aligned}$$

The lower limit of integration over β is not affected by the θ -function in the integrand of the R.H.S. of this equation and can be put equal to zero. The integral over β is convergent in this lower limit when the parameter s_Λ goes to infinity (there is, however, the usual infrared divergence at $\beta = 0$, $\epsilon = 0$, but it is irrelevant for the separation of the QMRK and MRK contributions determined by the parameter s_Λ). As for the upper limit of the integration, we easily get

$$\beta_{max} = 1 - \frac{(\vec{k}_1 - \vec{q}_1)^2}{s_\Lambda}. \quad (5.12)$$

There are two factors \tilde{I}_3 and I_4 in front of the two color structures $C_{3AA'}^{cc'}$ and $C_{4AA'}^{cc'}$, respectively, in Eq. (5.11). The first is

$$\begin{aligned}
\tilde{I}_3 = 2g^4 \int \int_0^{\beta_{max}} & \frac{\beta^{2\epsilon-1} d\beta}{(1 - \beta)} \frac{d^{D-2} k_1}{(2\pi)^{D-1}} \left[(\beta^2 + (1 - \beta)^2) (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} \right. \\
& \left. + 2\beta(1 - \beta) e_{A\mu}^\perp e_{A'\nu}^{*\perp} - 2\beta(1 - \beta) (1 - (1 + \epsilon)\beta(1 - \beta)) e_{A'\mu}^{*\perp} e_{A\nu}^\perp \right] R_{12\perp}'^\mu R_{12\perp}^\nu, \quad (5.13)
\end{aligned}$$

while the second can be obtained from \tilde{I}_3 by the replacement

$$R_{12\perp}' \rightarrow R_{21\perp}', \quad \beta_{max} \rightarrow 1. \quad (5.14)$$

Notice that the second of the substitutions (5.14) is possible because the integration over β in the expression for I_4 is convergent. Let us firstly consider the factor \tilde{I}_3 . In this case the integration over β can be carried out without any difficulties (see Eqs. (5.10) and (5.13)) and leads to

$$\begin{aligned}
\tilde{I}_3 = 2g^4 \int & \frac{d^{D-2} k_1}{(2\pi)^{D-1}} \left[\left(\ln \left(\frac{s_\Lambda}{(\vec{k}_1 - \vec{q}_1)^2} \right) + \frac{(1 - 2\epsilon)}{2\epsilon(1 + 2\epsilon)} + \psi(1) - \psi(1 + 2\epsilon) \right) \right. \\
& \times (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} + \frac{2}{(1 + 2\epsilon)} e_{A\mu}^\perp e_{A'\nu}^{*\perp} - \frac{(5 + 2\epsilon)}{(1 + 2\epsilon)(3 + 2\epsilon)} e_{A'\mu}^{*\perp} e_{A\nu}^\perp \left. \right] R_{12\perp}'^\mu R_{12\perp}^\nu. \quad (5.15)
\end{aligned}$$

Now we should also include the counterterm given by the second term in the R.H.S. of Eq. (2.9) into the two-gluon contribution to the impact factors, as it was already mentioned in Section 2. Using Eqs. (2.13) and (3.12), it is easy to check the following relation for the counterterm, taken with the minus sign which appears in Eq. (2.9):

$$\text{counterterm} = C_{3AA'}^{cc'} 2g^4 \int \frac{d^{D-2} k_1}{(2\pi)^{D-1}} \left[-\frac{1}{2} \ln \left(\frac{s_\Lambda^2}{s_0(\vec{k}_1 - \vec{q}_1)^2} \right) (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} \right] R_{12\perp}'^\mu R_{12\perp}^\nu. \quad (5.16)$$

Therefore, we can make the redefinition $\tilde{I}_3 \rightarrow I_3$ in order to include the counterterm contribution:

$$\tilde{I}_3 \rightarrow I_3 = 2g^4 \int \frac{d^{D-2} k_1}{(2\pi)^{D-1}} \left[\left(\frac{1}{2} \ln \left(\frac{s_0}{(\vec{k}_1 - \vec{q}_1)^2} \right) + \frac{(1 - 2\epsilon)}{2\epsilon(1 + 2\epsilon)} + \psi(1) - \psi(1 + 2\epsilon) \right) \right.$$

$$\times (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} + \frac{2}{(1+2\epsilon)} e_{A\mu}^\perp e_{A'\nu}^{*\perp} - \frac{(5+2\epsilon)}{(1+2\epsilon)(3+2\epsilon)} e_{A'\mu}^{*\perp} e_{A\nu}^\perp \Big] R_{12\perp}^{\prime\mu} R_{12\perp}^\nu . \quad (5.17)$$

Then the full expression for the two-gluon contribution to the gluon impact factors (2.9) does not depend on the artificial parameter s_Λ and takes the form

$$\Phi_{AA'}^{cc'(1)\{GG\}}(\vec{q}_1, \vec{q}_2, s_0) = C_{3AA'}^{cc'} I_3 + C_{4AA'}^{cc'} I_4 . \quad (5.18)$$

The next step to do is the calculation of I_3 . With help of Eqs. (5.10) and (5.17) we obtain

$$\begin{aligned} I_3 = & -\frac{1}{2} g^4 (e_{A'\perp}^* e_{A\perp}) \left[J_2(\vec{q}_1'^2) + J_2(\vec{q}_1^2) - \frac{4\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon}\Gamma(1+2\epsilon)} K_1 \right] \\ & - g^4 \left[\left(\ln \left(\frac{s_0}{\vec{q}^2} \right) + \frac{(1-2\epsilon)}{\epsilon(1+2\epsilon)} + 2\psi(1) - 2\psi(1+2\epsilon) \right) (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} \right. \\ & \left. + \frac{2}{(3+2\epsilon)} e_{A'\mu}^{*\perp} e_{A\nu}^\perp \right] \left[J_2^{\mu\nu}(q_{1\perp}') + J_2^{\mu\nu}(q_{1\perp}) - J_2^{\mu\nu}(q_\perp) \right] , \end{aligned} \quad (5.19)$$

with

$$\begin{aligned} J_2(\vec{v}^2) = & \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \ln \left(\frac{\vec{q}^2}{\vec{k}^2} \right) \frac{\vec{v}^2}{\vec{k}^2(\vec{k}-\vec{v})^2} = 2 \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \left[\frac{2}{\epsilon} \ln \left(\frac{\vec{q}^2}{\vec{v}^2} \right) \right. \\ & \left. \times (\vec{v}^2)^\epsilon + \frac{1}{\epsilon} \left(\frac{1}{\epsilon} + 2\psi(1-\epsilon) + 2\psi(1+2\epsilon) - 2\psi(1) - 2\psi(1+\epsilon) \right) (\vec{v}^2)^\epsilon \right] , \end{aligned} \quad (5.20)$$

$$K_1 = \frac{(4\pi)^{2+\epsilon}\Gamma(1+2\epsilon)}{4\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)} \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \ln \left(\frac{\vec{q}^2}{\vec{k}^2} \right) \frac{\vec{q}^2}{(\vec{k}-\vec{q}_1')^2(\vec{k}-\vec{q}_1)^2} , \quad (5.21)$$

$$\begin{aligned} J_2^{\mu\nu}(v_\perp) = & J_1^{\mu\nu} \left(v_\perp, \frac{m_f^2}{\beta^2} \right) \Big|_{m_f=0} = \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \\ & \times \left[\frac{1}{\epsilon(1+\epsilon)} (\vec{v}^2)^\epsilon g_{\perp\perp}^{\mu\nu} - \frac{1}{(1+\epsilon)(1+2\epsilon)} (\vec{v}^2)^\epsilon T_{\perp\perp}^{\mu\nu}(v_\perp) \right] . \end{aligned} \quad (5.22)$$

While the integrals J_2 and $J_1^{\mu\nu}$ (the second given in Eq. (4.17)) have been calculated exactly, K_1 is more complicated and we can calculate it only in the form of an expansion in ϵ . We do it in the Appendix. Substituting the exact results for J_2 and $J_2^{\mu\nu}$, I_3 becomes

$$\begin{aligned} I_3 = & -2g^4 \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} e_{A'\mu}^{*\perp} e_{A\nu}^\perp \left\{ g_{\perp\perp}^{\mu\nu} \left[F_2(\vec{q}_1'^2) + F_2(\vec{q}_1^2) - \frac{1}{\epsilon} \left(\ln \left(\frac{s_0}{\vec{q}^2} \right) + \frac{1}{\epsilon} - \frac{4}{(1+2\epsilon)} \right. \right. \right. \\ & \left. \left. \left. + \frac{1}{(1+\epsilon)(3+2\epsilon)} + 2\psi(1) - 2\psi(1+2\epsilon) \right) (\vec{q}^2)^\epsilon - K_1 \right] - \frac{1}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} \right. \\ & \left. \times \left[(\vec{q}_1'^2)^\epsilon T_{\perp\perp}^{\mu\nu}(q_{1\perp}') + (\vec{q}_1^2)^\epsilon T_{\perp\perp}^{\mu\nu}(q_{1\perp}) - (\vec{q}^2)^\epsilon T_{\perp\perp}^{\mu\nu}(q_\perp) \right] \right\} , \end{aligned} \quad (5.23)$$

where

$$\begin{aligned} F_2(\vec{v}^2) = & \frac{1}{\epsilon} \left(\ln \left(\frac{s_0}{\vec{v}^2} \right) + \frac{3}{2\epsilon} - \frac{4}{(1+2\epsilon)} + \frac{1}{(1+\epsilon)(3+2\epsilon)} \right. \\ & \left. + \psi(1) + \psi(1-\epsilon) - \psi(1+\epsilon) - \psi(1+2\epsilon) \right) (\vec{v}^2)^\epsilon . \end{aligned} \quad (5.24)$$

The calculation of I_4 is quite straightforward and can be performed starting from its definition through Eqs. (5.13) and (5.14). We obtain

$$I_4 = 2g^4 \int_0^1 \frac{\beta^{2\epsilon} d\beta}{\beta(1-\beta)} \left[(\beta^2 + (1-\beta)^2) (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} + 2\beta(1-\beta) e_{A\mu}^\perp e_{A'\nu}^{*\perp} \right. \\ \left. - 2\beta(1-\beta) (1 - (1+\epsilon)\beta(1-\beta)) e_{A'\mu}^{*\perp} e_{A\nu}^\perp \right] \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} R_{21\perp}'^\mu R_{12\perp}^\nu. \quad (5.25)$$

This equation, using Eqs. (5.10) and (5.22), leads to

$$I_4 = 2g^4 \int_0^1 \frac{d\beta}{\beta(1-\beta)} \left[(\beta^2 + (1-\beta)^2) (e_{A'\perp}^* e_{A\perp}) g_{\mu\nu}^{\perp\perp} + 2(1+\epsilon) (\beta(1-\beta))^2 e_{A'\mu}^{*\perp} e_{A\nu}^\perp \right] \\ \times \left[(1-\beta)^{2\epsilon} J_2^{\mu\nu}(q_{1\perp}') + \beta^{2\epsilon} J_2^{\mu\nu}(q_{1\perp}) - J_2^{\mu\nu}((1-\beta)q_{1\perp}' + \beta q_{1\perp}) \right], \quad (5.26)$$

which can be put in the form

$$I_4 = -2g^4 \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} e_{A'\mu}^{*\perp} e_{A\nu}^\perp \left[g_{\perp\perp}^{\mu\nu} \frac{(11+8\epsilon)}{\epsilon(1+2\epsilon)(3+2\epsilon)} \left((\vec{q}_1'^2)^\epsilon + (\vec{q}_1^2)^\epsilon \right) \right. \\ \left. - \frac{2}{(1+2\epsilon)(3+2\epsilon)} \left((\vec{q}_1'^2)^\epsilon \frac{q_{1\perp}'^\mu q_{1\perp}'^\nu}{q_{1\perp}'^2} + (\vec{q}_1^2)^\epsilon \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} \right) \right. \\ \left. + g_{\perp\perp}^{\mu\nu} \left(\frac{2}{\epsilon} K_2 - \frac{4}{\epsilon} K_3 + \frac{2(1+\epsilon)}{\epsilon(1+2\epsilon)} K_4 \right) + \frac{4(1+\epsilon)}{(1+2\epsilon)} K_{4\perp\perp}^{\mu\nu} \right]. \quad (5.27)$$

The integrals

$$K_2 = \int_0^1 \frac{d\beta}{\beta(1-\beta)} \left(((1-\beta)\vec{q}_1' + \beta\vec{q}_1)^2 \right)^\epsilon - (1-\beta)^{2\epsilon} (\vec{q}_1'^2)^\epsilon - \beta^{2\epsilon} (\vec{q}_1^2)^\epsilon, \\ K_3 = \int_0^1 d\beta \left(((1-\beta)\vec{q}_1' + \beta\vec{q}_1)^2 \right)^\epsilon, \quad K_4 = \int_0^1 d\beta \beta(1-\beta) \left(((1-\beta)\vec{q}_1' + \beta\vec{q}_1)^2 \right)^\epsilon, \\ K_{4\perp\perp}^{\mu\nu} = \int_0^1 d\beta \beta(1-\beta) \left(((1-\beta)\vec{q}_1' + \beta\vec{q}_1)^2 \right)^\epsilon \frac{((1-\beta)q_{1\perp}' + \beta q_{1\perp})_\perp^\mu ((1-\beta)q_{1\perp}' + \beta q_{1\perp})_\perp^\nu}{((1-\beta)q_{1\perp}' + \beta q_{1\perp})_\perp^2}, \quad (5.28)$$

which appear in Eq. (5.27), cannot be calculated at arbitrary ϵ . We evaluate them in the Appendix in form of an expansion in ϵ . Let us note that the evident relation

$$K_4 = K_{4\perp\mu}^{\mu\perp}, \quad (5.29)$$

is not very useful for us, because these two integrals should be calculated with different accuracy (see factors in front of these integrals in Eq. (5.27)), so that we are forced to consider them separately.

At this point the calculation of the two-gluon contribution to the gluon impact factors defined in Eq. (2.9) is completed. The two-gluon contribution is given by the relation (5.18) with the color structures $C_{3AA'}^{cc'}$ and $C_{4AA'}^{cc'}$ shown in Eq. (5.4) and the coefficients I_3 and I_4 presented in Eqs. (5.23) and (5.27), respectively.

To obtain the impact factors $\Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1, \vec{q}; s_0)$ with the t -channel state ν of the irreducible representation \mathcal{R} of the color group $SU(N)$, it is enough to perform the convolution in Eq. (2.8).

6 The check of the bootstrap

We have now all the contributions needed to check the bootstrap condition given in Eq. (2.7). First of all, we must consider the gluon non-forward impact factors in a generic state a of the octet color representation in the t -channel. According to Eq. (2.8), this means that the four one-loop order contributions to the unprojected gluon impact factors, due to one-gluon, quark-antiquark and two-gluon intermediate states given in Eqs. (3.13), (4.6) and (5.11) respectively, and to the counterterm given in Eq. (5.16) must be contracted with

$$\langle cc' | \hat{\mathcal{P}}_8 | a \rangle = \frac{i T_{cc'}^a}{\sqrt{N}}. \quad (6.1)$$

Since the projection on the octet color state a gives

$$\begin{aligned} i\sqrt{N} \langle cc' | \hat{\mathcal{P}}_8 | a \rangle (T^{c'} T^c)_{A'A} &= \frac{N}{2} T_{A'A}^a, & i\sqrt{N} \langle cc' | \hat{\mathcal{P}}_8 | a \rangle C_{1AA'}^{cc'} &= \frac{N}{4} T_{A'A}^a, \\ i\sqrt{N} \langle cc' | \hat{\mathcal{P}}_8 | a \rangle C_{3AA'}^{cc'} &= \frac{N^2}{4} T_{A'A}^a, & i\sqrt{N} \langle cc' | \hat{\mathcal{P}}_8 | a \rangle C_{2AA'}^{cc'} &= i\sqrt{N} \langle cc' | \hat{\mathcal{P}}_8 | a \rangle C_{4AA'}^{cc'} = 0, \end{aligned} \quad (6.2)$$

we obtain for the octet gluon impact factors

$$\begin{aligned} i\sqrt{N} \Phi_{A'A}^{(8,a)(1)}(\vec{q}_1, \vec{q}; s_0) &= i\sqrt{N} \langle cc' | \hat{\mathcal{P}}_8 | a \rangle \Phi_{AA'}^{cc'(1)}(\vec{q}_1, \vec{q}; s_0) = -\frac{1}{4} T_{A'A}^a g^2 N (e_{A'\perp}^* e_{A\perp}) \omega^{(1)}(-\vec{q}^2) \\ &\times \ln\left(\frac{s_0}{\vec{q}^2}\right) + T_{A'A}^a g^4 N^2 \frac{\Gamma(1-\epsilon)}{2(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} e_{A'\mu}^* e_{A\nu}^\perp \left[g_{\perp\perp}^{\mu\nu} \frac{1}{\epsilon} \left(\frac{1}{\epsilon} - \frac{4}{(1+2\epsilon)} + \frac{1}{(1+\epsilon)(3+2\epsilon)} \right. \right. \\ &\quad \left. \left. + 2\psi(1) - 2\psi(1+2\epsilon) \right) (\vec{q}^2)^\epsilon - \frac{1}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} T_{\perp\perp}^{\mu\nu}(q_\perp) (\vec{q}^2)^\epsilon \right] + T_{A'A}^a g^4 N \\ &\times \frac{\Gamma(1-\epsilon)}{4(1+\epsilon)(4\pi)^{2+\epsilon}} e_{A'\mu}^* e_{A\nu}^\perp \left[g_{\perp\perp}^{\mu\nu} F_1^{(+)}(\vec{q}^2) + 4T_{\perp\perp}^{\mu\nu}(q_\perp) F_1^{(-)}(\vec{q}^2) \right] + T_{A'A}^a g^4 N^2 \frac{\Gamma(1-\epsilon)}{2(4\pi)^{2+\epsilon}} \\ &\times \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} (e_{A'\perp}^* e_{A\perp}) \left[\frac{1}{2\epsilon} \left(\frac{1}{\epsilon} + \frac{(11+7\epsilon)}{(1+2\epsilon)(3+2\epsilon)} + 2\psi(1+2\epsilon) - 2\psi(1+\epsilon) \right) \right. \\ &\quad \left. \times \left((\vec{q}_1'^2)^\epsilon + (\vec{q}_1^2)^\epsilon \right) + K_1 \right] - T_{A'A}^a g^4 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} (e_{A'\perp}^* e_{A\perp}) \frac{1}{\epsilon} \\ &\times \sum_f \int_0^1 dx x(1-x) \left[(m_f^2 + x(1-x)\vec{q}_1'^2)^\epsilon + (m_f^2 + x(1-x)\vec{q}_1^2)^\epsilon \right], \end{aligned} \quad (6.3)$$

the one-loop Reggeized gluon trajectory $\omega^{(1)}$ being given in Eq. (3.11), the tensor $T_{\perp\perp}^{\mu\nu}$ in Eq. (3.5), the functions $F_1^{(\pm)}$ in Eq. (3.9) and the integral K_1 in Eq. (5.21). Using the integral representation (3.11) for the one-loop gluon Regge trajectory, we get for the L.H.S. of the bootstrap condition (2.7)

$$\text{L.H.S.} = -\frac{1}{2} T_{A'A}^a g (e_{A'\perp}^* e_{A\perp}) \left(\omega^{(1)}(-\vec{q}^2) \right)^2 \ln\left(\frac{s_0}{\vec{q}^2}\right) + T_{A'A}^a g^3 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}$$

$$\begin{aligned}
& \times e_{A'\mu}^{*\perp} e_{A\nu}^{\perp} \omega^{(1)}(-\vec{q}^2) \left[g_{\perp\perp}^{\mu\nu} \frac{1}{\epsilon} \left(\frac{1}{\epsilon} - \frac{4}{(1+2\epsilon)} + \frac{1}{(1+\epsilon)(3+2\epsilon)} + 2\psi(1) - 2\psi(1+2\epsilon) \right) (\vec{q}^2)^\epsilon \right. \\
& \quad \left. - \frac{1}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} T_{\perp\perp}^{\mu\nu}(q_\perp) (\vec{q}^2)^\epsilon \right] + T_{A'A}^a g^3 \frac{\Gamma(1-\epsilon)}{2(1+\epsilon)(4\pi)^{2+\epsilon}} e_{A'\mu}^{*\perp} e_{A\nu}^{\perp} \omega^{(1)}(-\vec{q}^2) \\
& \quad \times \left[g_{\perp\perp}^{\mu\nu} F_1^{(+)}(\vec{q}^2) + 4T_{\perp\perp}^{\mu\nu}(q_\perp) F_1^{(-)}(\vec{q}^2) \right] - T_{A'A}^a g^5 N^2 \frac{\Gamma(1-\epsilon)}{2(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} (e_{A'\perp}^* e_{A\perp}) \\
& \times \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{q}_1^2(\vec{q}_1 - \vec{q})^2} \left[\frac{1}{\epsilon} \left(\frac{1}{\epsilon} + \frac{(11+7\epsilon)}{(1+2\epsilon)(3+2\epsilon)} + 2\psi(1+2\epsilon) - 2\psi(1+\epsilon) \right) (\vec{q}_1^2)^\epsilon + K_1 \right] \\
& \quad + 2T_{A'A}^a g^5 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} (e_{A'\perp}^* e_{A\perp}) \frac{1}{\epsilon} \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{q}_1^2(\vec{q}_1 - \vec{q})^2} \\
& \quad \times \sum_f \int_0^1 dx x(1-x) (m_f^2 + x(1-x)\vec{q}_1^2)^\epsilon . \tag{6.4}
\end{aligned}$$

The analogous expression for the R.H.S. of Eq. (2.7) takes the form

$$\begin{aligned}
\text{R.H.S.} = & -\frac{1}{2} T_{A'A}^a g (e_{A'\perp}^* e_{A\perp}) \omega^{(2)}(-\vec{q}^2) - \frac{1}{2} T_{A'A}^a g (e_{A'\perp}^* e_{A\perp}) (\omega^{(1)}(-\vec{q}^2))^2 \ln \left(\frac{s_0}{\vec{q}^2} \right) \\
& - T_{A'A}^a g^3 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} e_{A'\mu}^{*\perp} e_{A\nu}^{\perp} \omega^{(1)}(-\vec{q}^2) \left[g_{\perp\perp}^{\mu\nu} \frac{1}{\epsilon} \left(-\frac{2}{\epsilon} + \frac{9(1+\epsilon)^2 + 2}{2(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} \right. \right. \\
& \quad \left. \left. + 2\psi(1+\epsilon) - \psi(1-\epsilon) - \psi(1) \right) (\vec{q}^2)^\epsilon + \frac{1}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} T_{\perp\perp}^{\mu\nu}(q_\perp) (\vec{q}^2)^\epsilon \right] \\
& + T_{A'A}^a g^3 \frac{\Gamma(1-\epsilon)}{2(1+\epsilon)(4\pi)^{2+\epsilon}} e_{A'\mu}^{*\perp} e_{A\nu}^{\perp} \omega^{(1)}(-\vec{q}^2) \left[g_{\perp\perp}^{\mu\nu} F_1^{(+)}(\vec{q}^2) + 4T_{\perp\perp}^{\mu\nu}(q_\perp) F_1^{(-)}(\vec{q}^2) \right] \\
& - 2T_{A'A}^a g^3 \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} (e_{A'\perp}^* e_{A\perp}) \frac{1}{\epsilon} \omega^{(1)}(-\vec{q}^2) \sum_f \int_0^1 dx x(1-x) (m_f^2 + x(1-x)\vec{q}^2)^\epsilon , \tag{6.5}
\end{aligned}$$

where $\omega^{(2)}$ is the two-loop correction to the Reggeized gluon trajectory and we have used Eqs. (3.2)-(3.5), (3.8) and (3.9) which give the gluon-gluon-Reggeon effective vertex with one-loop accuracy.

We see that the helicity non-conserving parts (the terms with the tensor $T_{\perp\perp}^{\mu\nu}$) are completely cancelled in both L.H.S. and R.H.S. of the bootstrap condition. This fact is very important, because up to now the possibility to present in the Regge form the helicity non-conserving part of the elastic scattering amplitude with gluon color quantum numbers in the t -channel was not checked anywhere.

As for the helicity conserving part, the bootstrap condition for it is not quite independent from the calculation of the two-loop correction to the gluon trajectory [4], since it was performed assuming that the gluon Reggeization holds, by comparison of the s -channel discontinuity dictated by the Regge form (2.4) with that calculated from the unitarity. Therefore, the check of the bootstrap condition for the helicity conserving part gives more a test of correctness of all the calculations involved in the determination of the trajectory and of the impact factors.

We notice that there are cancellations between the terms in Eq. (6.4) and (6.5) containing the factors $\ln(s_0/\vec{q}^2)$ and the functions $F_1^{(\pm)}$, respectively. Then we arrive at the following equality, which must be valid for the bootstrap:

$$\begin{aligned}
\omega^{(2)}(-\vec{q}^2) &= \omega^{(2)(G)}(-\vec{q}^2) + \omega^{(2)(Q)}(-\vec{q}^2) = g^4 N^2 \frac{\Gamma(1-\epsilon)}{2(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \\
&\times \frac{\vec{q}^2}{\vec{q}_1^2(\vec{q}_1 - \vec{q})^2} \left[\frac{1}{\epsilon} \left(\frac{1}{\epsilon} + \frac{(11+7\epsilon)}{(1+2\epsilon)(3+2\epsilon)} + 2\psi(1+2\epsilon) - 2\psi(1+\epsilon) \right) \left(2(\vec{q}_1^2)^\epsilon - (\vec{q}^2)^\epsilon \right) \right. \\
&- \frac{1}{\epsilon} \left(\frac{1}{\epsilon} + 2\psi(1+2\epsilon) - 2\psi(1+\epsilon) + 2\psi(1-\epsilon) - 2\psi(1) \right) (\vec{q}^2)^\epsilon + 2K_1 \Big] + 2g^4 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{1}{\epsilon} \\
&\times \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{q}_1^2(\vec{q}_1 - \vec{q})^2} \sum_f \int_0^1 dx x(1-x) \\
&\times \left[\left(m_f^2 + x(1-x)\vec{q}^2 \right)^\epsilon - 2 \left(m_f^2 + x(1-x)\vec{q}_1^2 \right)^\epsilon \right]. \tag{6.6}
\end{aligned}$$

Here $\omega^{(2)(G)}$ and $\omega^{(2)(Q)}$ are the gluon and quark contributions to the two-loop correction to the gluon Regge trajectory $\omega^{(2)}$; they are known and can be found, for example, in Ref. [20]. The cancellation of the quark contribution in both sides of Eq. (6.6) is evident if one compares this equation with the expression (71) of Ref. [20] for $\omega^{(2)(Q)}$. Let us check the cancellation of the gluon contribution. To do this, consider the integral representation of Ref. [20] for the gluon part $\omega^{(2)(G)}$ of the two-loop correction to the gluon trajectory:

$$\begin{aligned}
\omega^{(2)(G)}(-\vec{q}^2) &= \frac{g^4 N^2}{2} \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{d^{D-2}q_2}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{q}_1^2 \vec{q}_2^2} \left[\frac{\vec{q}^2}{2(\vec{q}_1 - \vec{q})^2(\vec{q}_2 - \vec{q})^2} \ln \left(\frac{\vec{q}^2}{(\vec{q}_1 - \vec{q}_2)^2} \right) \right. \\
&- \frac{1}{(\vec{q}_1 + \vec{q}_2 - \vec{q})^2} \ln \left(\frac{(\vec{q}_1 - \vec{q})^2}{\vec{q}_1^2} \right) + \left(\frac{-\vec{q}^2}{2(\vec{q}_1 - \vec{q})^2(\vec{q}_2 - \vec{q})^2} + \frac{1}{(\vec{q}_1 + \vec{q}_2 - \vec{q})^2} \right) \\
&\times \left. \left(\frac{1}{\epsilon} + \frac{(11+7\epsilon)}{2(1+2\epsilon)(3+2\epsilon)} + 2\psi(1+2\epsilon) - 2\psi(1+\epsilon) + \psi(1-\epsilon) - \psi(1) \right) \right]. \tag{6.7}
\end{aligned}$$

It can be written in the form

$$\begin{aligned}
\omega^{(2)(G)}(-\vec{q}^2) &= \frac{g^4 N^2}{2} \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{q}_1^2(\vec{q}_1 - \vec{q})^2} \left[\frac{1}{2} \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \ln \left(\frac{\vec{q}^2}{\vec{k}^2} \right) \frac{\vec{q}^2}{(\vec{k} - \vec{q}_1')^2(\vec{k} - \vec{q}_1)^2} - \right. \\
&\int \frac{d^{D-2}k}{(2\pi)^{D-1}} \ln \left(\frac{\vec{q}_1^2}{\vec{k}^2} \right) \frac{\vec{q}_1^2}{\vec{k}^2(\vec{k} - \vec{q}_1)^2} + \frac{1}{2} \left(2 \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \frac{\vec{q}_1^2}{\vec{k}^2(\vec{k} - \vec{q}_1)^2} - \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{k}^2(\vec{k} - \vec{q})^2} \right) \\
&\times \left. \left(\frac{1}{\epsilon} + \frac{(11+7\epsilon)}{2(1+2\epsilon)(3+2\epsilon)} + 2\psi(1+2\epsilon) - 2\psi(1+\epsilon) + \psi(1-\epsilon) - \psi(1) \right) \right]. \tag{6.8}
\end{aligned}$$

Let us now express the first integral into square brackets of this equation through K_1 , given in Eq. (5.21), and calculate exactly the other integrals, making use of the relations (3.11) and (5.20). So doing, we get just the first term in the R.H.S. of Eq. (6.6), what proves that the bootstrap condition (2.7) is satisfied as far as the gluon NLA impact factors are concerned.

To summarize, we have verified the fulfillment of the bootstrap conditions for the helicity non-conserving and conserving parts, separately. We stress that, while for the helicity non-conserving part of Eq. (2.7) it gives the first check of the possibility to present it in the Regge form, for the helicity conserving part it represents also a check of the previous result of Ref. [4] for the two-loop Reggeized gluon trajectory $\omega^{(2)}$. So, at least the integral representation for $\omega^{(2)}$ obtained there is correct. But since there is also an independent check [16] of the integrated result of Ref. [5] for $\omega^{(2)}$, the final result of that paper is fully verified.

7 Gluon impact factors in massless QCD

In this section we calculate explicitly the gluon NLA non-forward impact factors (2.9), restricting ourselves to the case of massless quarks. In this case most integrations can be carried out exactly at arbitrary ϵ and there are only few integrals which we are forced to calculate as expansion in ϵ . The Born contribution to the impact factor, which we present here for the sake of completeness, is not changed, of course; it is (see Eq. (3.12))

$$\Phi_{AA'}^{cc'(B)}(\vec{q}_1, \vec{q}) = -g^2 (T^{c'} T^c)_{A'A} e_{A'\perp}^{*\mu} e_{A\perp}^\nu g_{\mu\nu}^{\perp\perp}. \quad (7.1)$$

The integration in Eq. (3.13), as well as the integrations in Eqs. (3.9), which give the functions $F_1^{(\pm)}$ can be easily done, leading to

$$\begin{aligned} \sum_f \frac{2(1+\epsilon)}{\epsilon} \int_0^1 dx x(1-x) (m_f^2 + x(1-x)\vec{v}^2)^\epsilon \Big|_{m_f=0} \\ = \frac{n_f \Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \frac{(1+\epsilon)^2}{\epsilon(1+2\epsilon)(3+2\epsilon)} (\vec{v}^2)^\epsilon, \end{aligned} \quad (7.2)$$

$$\begin{aligned} F_1^{(+)}(\vec{v}^2) \Big|_{m_f=0} &= \frac{n_f \Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \frac{2}{\epsilon} \left(\frac{(1+\epsilon)}{(1+2\epsilon)} - \frac{1}{(1+\epsilon)(3+2\epsilon)} \right) (\vec{v}^2)^\epsilon, \\ F_1^{(-)}(\vec{v}^2) \Big|_{m_f=0} &= \frac{n_f \Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \frac{1}{2(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} (\vec{v}^2)^\epsilon, \end{aligned} \quad (7.3)$$

where n_f is the number of light quark flavors. Then from Eq. (3.13), with the help of last three relations and of Eq. (3.5), we get the one-gluon contribution to the one-loop correction for the gluon impact factors (2.9) in the massless quark case:

$$\begin{aligned} \Phi_{AA'}^{cc'(1)\{G\}}(\vec{q}_1, \vec{q}; s_0) \left(g^4 N^2 \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \right)^{-1} &= \frac{1}{N} (T^{c'} T^c)_{A'A} e_{A'\mu}^{*\perp} e_{A\nu}^\perp \\ &\times \left[g_{\perp\perp}^{\mu\nu} \frac{1}{\epsilon} \left(\ln \left(\frac{s_0}{\vec{q}_1'^2} \right) (\vec{q}_1'^2)^\epsilon + \ln \left(\frac{s_0}{\vec{q}_1^2} \right) (\vec{q}_1^2)^\epsilon \right) + g_{\perp\perp}^{\mu\nu} \frac{1}{\epsilon} \left(\frac{2}{\epsilon} - \frac{(11+9\epsilon)}{2(1+2\epsilon)(3+2\epsilon)} \right) \right. \\ &\left. + \frac{n_f}{N} \frac{(1+\epsilon)(2+\epsilon)-1}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} + \psi(1) + \psi(1-\epsilon) - 2\psi(1+\epsilon) \right] \left((\vec{q}_1'^2)^\epsilon + (\vec{q}_1^2)^\epsilon \right) \end{aligned}$$

$$+ \frac{2}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} \left(1 + \epsilon - \frac{n_f}{N} \right) \left(\frac{q_{1\perp}'^\mu q_{1\perp}'^\nu}{q_{1\perp}'^2} (\vec{q}_1'^2)^\epsilon + \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} (\vec{q}_1^2)^\epsilon \right) \Big]. \quad (7.4)$$

For the case of the quark-antiquark intermediate state contribution, all integrations in Eq. (4.21) (see for instance Eqs. (7.3)) and the integration over the variable x in Eq. (4.24) can be easily performed in exact form. As for integration over the variable β in (4.24), it cannot be completely carried out for arbitrary ϵ . With help of the Eqs. (3.5), (4.7), (4.13), (4.21), (4.24) and (7.3) we obtain

$$\begin{aligned} \Phi_{AA'}^{cc'(1)\{q\bar{q}\}}(\vec{q}_1, \vec{q}) & \left(g^4 N^2 \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \right)^{-1} = \frac{2}{N} \text{tr} \left(t^A t^c t^{c'} t^{A'} + t^A t^{A'} t^{c'} t^c \right) e_{A'\mu}^{*\perp} e_{A\nu}^\perp \frac{n_f}{N} \\ & \times \left[-g_{\perp\perp}^{\mu\nu} \frac{2(1+\epsilon)^2 + \epsilon}{\epsilon(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} \left((\vec{q}_1'^2)^\epsilon + (\vec{q}_1^2)^\epsilon - (\vec{q}^2)^\epsilon \right) + \frac{2}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} \right. \\ & \times \left(\frac{q_{1\perp}'^\mu q_{1\perp}'^\nu}{q_{1\perp}'^2} (\vec{q}_1'^2)^\epsilon + \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} (\vec{q}_1^2)^\epsilon - \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} (\vec{q}^2)^\epsilon \right) \Big] + \frac{2}{N} \text{tr} \left(t^A t^c t^{A'} t^{c'} + t^A t^{c'} t^{A'} t^c \right) e_{A'\mu}^{*\perp} e_{A\nu}^\perp \\ & \times \frac{n_f}{N} \left[g_{\perp\perp}^{\mu\nu} \frac{(2+\epsilon)}{\epsilon(1+\epsilon)(3+2\epsilon)} \left((\vec{q}_1'^2)^\epsilon + (\vec{q}_1^2)^\epsilon \right) - \frac{2}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} \left(\frac{q_{1\perp}'^\mu q_{1\perp}'^\nu}{q_{1\perp}'^2} (\vec{q}_1'^2)^\epsilon \right. \right. \\ & \quad \left. \left. + \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} (\vec{q}_1^2)^\epsilon \right) - g_{\perp\perp}^{\mu\nu} \left(\frac{1}{\epsilon} K_3 - \frac{2}{\epsilon(1+2\epsilon)} K_4 \right) + \frac{4}{(1+2\epsilon)} K_{4\perp\perp}^{\mu\nu} \right], \quad (7.5) \end{aligned}$$

with the integrals K_3 , K_4 and $K_{4\perp\perp}^{\mu\nu}$ presented in the Eqs. (5.28).

As for the NLA two-gluon contribution to the gluon impact factors for the non-forward scattering, found in Section 5, it does not change in the massless quark case. For completeness we write down its expression below with help of Eqs. (3.5), (5.4), (5.18), (5.23), (5.24) and (5.27):

$$\begin{aligned} \Phi_{AA'}^{cc'(1)\{GG\}}(\vec{q}_1, \vec{q}; s_0) & \left(g^4 N^2 \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \right)^{-1} = -\frac{2}{N^2} (T^{c'_1} T^{c_1})_{A'A} (T^{c'_1} T^{c_1})_{c'c} \\ & \times e_{A'\mu}^{*\perp} e_{A\nu}^\perp \left[g_{\perp\perp}^{\mu\nu} \frac{1}{\epsilon} \left(\ln \left(\frac{s_0}{\vec{q}_1'^2} \right) (\vec{q}_1'^2)^\epsilon + \ln \left(\frac{s_0}{\vec{q}_1^2} \right) (\vec{q}_1^2)^\epsilon - \ln \left(\frac{s_0}{\vec{q}^2} \right) (\vec{q}^2)^\epsilon \right) + g_{\perp\perp}^{\mu\nu} \frac{1}{\epsilon} \left(\frac{3}{2\epsilon} \right. \right. \\ & \quad \left. \left. - \frac{(11+8\epsilon)}{(1+2\epsilon)(3+2\epsilon)} - \psi(1+2\epsilon) - \psi(1+\epsilon) + \psi(1-\epsilon) + \psi(1) \right) \left((\vec{q}_1'^2)^\epsilon + (\vec{q}_1^2)^\epsilon - (\vec{q}^2)^\epsilon \right) \right. \\ & \quad \left. + g_{\perp\perp}^{\mu\nu} \frac{1}{2\epsilon} \left(\frac{1}{\epsilon} + 2\psi(1+2\epsilon) - 2\psi(1+\epsilon) + 2\psi(1-\epsilon) - 2\psi(1) \right) (\vec{q}^2)^\epsilon - g_{\perp\perp}^{\mu\nu} K_1 + \frac{2}{(1+2\epsilon)(3+2\epsilon)} \right. \\ & \quad \times \left(\frac{q_{1\perp}'^\mu q_{1\perp}'^\nu}{q_{1\perp}'^2} (\vec{q}_1'^2)^\epsilon + \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} (\vec{q}_1^2)^\epsilon - \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} (\vec{q}^2)^\epsilon \right) \Big] - \frac{2}{N^2} (T^{c'_1} T^{c_1})_{c'A} (T^{c'_1} T^{c_1})_{A'c} e_{A'\mu}^{*\perp} e_{A\nu}^\perp \\ & \times \left[g_{\perp\perp}^{\mu\nu} \frac{(11+8\epsilon)}{\epsilon(1+2\epsilon)(3+2\epsilon)} \left((\vec{q}_1'^2)^\epsilon + (\vec{q}_1^2)^\epsilon \right) - \frac{2}{(1+2\epsilon)(3+2\epsilon)} \left(\frac{q_{1\perp}'^\mu q_{1\perp}'^\nu}{q_{1\perp}'^2} (\vec{q}_1'^2)^\epsilon + \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} \right. \right. \\ & \quad \left. \left. \times (\vec{q}_1^2)^\epsilon \right) + g_{\perp\perp}^{\mu\nu} \left(\frac{2}{\epsilon} K_2 - \frac{4}{\epsilon} K_3 + \frac{2(1+\epsilon)}{\epsilon(1+2\epsilon)} K_4 \right) + \frac{4(1+\epsilon)}{(1+2\epsilon)} K_{4\perp\perp}^{\mu\nu} \right], \quad (7.6) \end{aligned}$$

with the integrals K_1 and K_2 defined by Eqs. (5.21) and (5.28), respectively. So we see that, to obtain the gluon impact factors (2.9) in the massless quark case, we should calculate the integrals $K_1 - K_4$ and $K_{4\perp\perp}^{\mu\nu}$. This calculation can be done only in the form of an expansion in ϵ , as it was already mentioned above, and we perform it in the Appendix. In order to understand what is the accuracy according to which the integrals must be calculated, we notice that further applications of the non-forward impact factors imply a subsequent integration over \vec{q}_1 in the form like that in the bootstrap condition (2.7) (see Ref. [14]). In this subsequent integration the integrand is singular in the regions of $\vec{q}_1 \rightarrow 0$ and $\vec{q}_1' = \vec{q}_1 - \vec{q} \rightarrow 0$, so that in these limits one must have exact expressions for the impact factors (or, at least, expressions which lead to an accuracy up to finite terms in the physical limit $\epsilon \rightarrow 0$ after the integration over \vec{q}_1). An analogous situation was observed in the calculation of the forward BFKL equation kernel in Ref. [7] and detailed explanations can be found there. In the regions where the integrand is non-singular, it is enough to know impact factors with accuracy up to terms of the type ϵ^0 . The above discussion, taking also into account the coefficients in the integrals $K_1 - K_4$ and $K_{4\perp\perp}^{\mu\nu}$ given by the expressions (7.5) and (7.6) for the corresponding contributions to the impact factors, make clear what terms we should keep in the expansion in ϵ for each of these integrals.

For the case of the forward scattering, being

$$\vec{q} = 0, \quad \vec{q}_1' = \vec{q}_1, \quad (7.7)$$

we have further simplifications, which lead to the possibility to calculate the integrals $K_1 - K_4$ and $K_{4\perp\perp}^{\mu\nu}$ in exact form without expansion in ϵ ; we find

$$\begin{aligned} K_1 &= 0, \quad K_2 = \left(-\frac{1}{\epsilon} + 2\psi(1 + 2\epsilon) - 2\psi(1) \right) (\vec{q}_1^2)^\epsilon, \\ K_3 &= (\vec{q}_1^2)^\epsilon, \quad K_4 = \frac{1}{6} (\vec{q}_1^2)^\epsilon, \quad K_{4\perp\perp}^{\mu\nu} = \frac{1}{6} \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} (\vec{q}_1^2)^\epsilon. \end{aligned} \quad (7.8)$$

Let us stress that the results for these integrals shown in the Appendix for the non-forward case do not have the correct asymptotics as in the forward one and can be used only for $\vec{q} \neq 0$. We did not care in the Appendix about it, because these two cases are well separated and also because the integrals for the forward scattering are calculated exactly. For example, we consider here the expression for the forward gluon impact factor with the singlet color state in the t -channel, putting also the helicity $\lambda_{A'}$ to be equal to λ_A and taking an average over this quantum number. Consequently, the Born contribution reads

$$\Phi_G^{(0)(B)} = g^2 \sqrt{\frac{N^2}{N^2 - 1}} \quad (7.9)$$

and the one-loop one takes the form

$$\begin{aligned} \Phi_G^{(0)(1)}(\vec{q}_1; s_0) &= \Phi_G^{(0)(B)} \omega^{(1)}(-\vec{q}_1^2) \left[-\ln \left(\frac{s_0}{\vec{q}_1^2} \right) + \frac{11}{6} - \frac{2\epsilon}{(1 + 2\epsilon)(3 + 2\epsilon)} + \psi(1) \right. \\ &\quad \left. - \psi(1 - \epsilon) + \left(\frac{11}{3} - \frac{2}{3} \frac{n_f}{N} \right) \frac{3(1 + \epsilon)}{2(1 + 2\epsilon)(3 + 2\epsilon)} + \frac{n_f}{N^3} \frac{(2 + 3\epsilon)}{6(1 + \epsilon)} \right], \end{aligned} \quad (7.10)$$

where the one-loop Reggeized gluon trajectory $\omega^{(1)}$ is defined by Eq. (3.11). In these two last equations we also have used the relation (see Eq. (2.8))

$$\langle cc' | \hat{\mathcal{P}}_0 | 0 \rangle = \frac{\delta_{cc'}}{\sqrt{N^2 - 1}} \quad (7.11)$$

and have omitted the common color factor $\delta_{AA'}$.

The gluon impact factor in the forward case ($t = 0$ and color singlet in the t -channel) was considered in Refs. [22, 23]. In Ref. [23] it was calculated for massless quarks with accuracy up to terms finite in the $\epsilon \rightarrow 0$ limit. Unfortunately, the comparison of our result (7.10) for this particular case with the corresponding result of Ref. [23] is not straightforward because of the different definitions adopted. First of all, we have used up to now a fixed energy scale s_0 independent of the virtualities of the Reggeized gluons. The transition to the general case of any factorizable scale $s_0 = \sqrt{f_1(\vec{q}_1, \vec{q}) f_2(\vec{q}_2, \vec{q})}$ in Eq. (2.5) can be made to the NLA accuracy without changing the Green function by the substitution [13]

$$\begin{aligned} & \Phi_{A'A}^{(\mathcal{R}, \nu)}(\vec{q}_1, \vec{q}; s_0) \longrightarrow \\ & \longrightarrow \Phi_{A'A}^{(\mathcal{R}, \nu)}(\vec{q}_1, \vec{q}; s_0) + \frac{1}{2} \int \frac{d^{D-2} q_r}{\vec{q}_r^2 \vec{q}'^2} \Phi_{A'A}^{(\mathcal{R}, \nu)(B)}(\vec{q}_r, \vec{q}) \mathcal{K}^{(\mathcal{R})(B)}(\vec{q}_r, \vec{q}_1; \vec{q}) \ln \left(\frac{f_1(\vec{q}_r, \vec{q})}{s_0} \right), \end{aligned} \quad (7.12)$$

where $\mathcal{K}^{(\mathcal{R})(B)}$ is the non-forward BFKL kernel in the LLA [1, 12, 14]

$$\begin{aligned} \mathcal{K}^{(\mathcal{R})(B)}(\vec{q}_1, \vec{q}_2, \vec{q}) &= \vec{q}_1^2 \vec{q}_1'^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) \left(\omega^{(1)}(-\vec{q}_1^2) + \omega^{(1)}(-\vec{q}_1'^2) \right) \\ &+ \frac{g^2}{(2\pi)^{D-1}} c_{\mathcal{R}} \left(\frac{\vec{q}_1^2 \vec{q}_2'^2 + \vec{q}_1'^2 \vec{q}_2^2}{(\vec{q}_1 - \vec{q}_2)^2} - \vec{q}^2 \right). \end{aligned} \quad (7.13)$$

The coefficient $c_{\mathcal{R}}$ is equal to N for the singlet and to $N/2$ for the octet representation. In the particular case of the forward helicity conserving impact factor and Regge-motivated scale $s_0 = |\vec{q}_1| |\vec{q}_2|$ used in Ref. [9], it changes our result (7.10) to the expression

$$\begin{aligned} & \frac{\Phi_G^{(0)(1)}(\vec{q}_1; s_0)}{\Phi_G^{(0)(B)}} \longrightarrow \omega^{(1)}(-\vec{q}_1^2) \left[\left(\frac{11}{6} - \frac{n_f}{3N} \right) + \left(\frac{11}{6} + \frac{(2+\epsilon)n_f}{6N} \right) - \frac{C_F n_f}{N^2} \left(\frac{2}{3} + \frac{\epsilon}{3} \right) \right. \\ & \left. - \frac{\epsilon}{N} \left(N \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5n_f}{9} \right) \right] + g^2 N \vec{q}_1^2 \frac{1}{(2\pi)^{D-1}} \int \frac{d^{D-2} q_r}{\vec{q}_r^2 (\vec{q}_r - \vec{q}_1)^2} \ln \left(\frac{\vec{q}_r^2}{\vec{q}_1^2} \right), \end{aligned} \quad (7.14)$$

where $C_F = (N^2 - 1)/(2N)$ and we have also made an expansion in ϵ in the integrated part with accuracy up to terms finite in the limit $\epsilon \rightarrow 0$. The scale $s_0 = |\vec{q}_1| |\vec{q}_2|$ was adopted also in [23], but the impact factors were defined there on the basis of their infrared properties, so that additional operator factors H_L and H_R were introduced [10, 23] in the Green function. Therefore, one has to compare our result with changed scale (7.14) not simply with the expression (5.11) of Ref. [23], but with this expression plus the piece connected with the operator H . To our opinion, there is misprint in the expression (3.12) of Ref. [23] for this operator, which should contain the additional factor $1/\Gamma(1 - \epsilon)$. If so, one can easily obtain from Ref. [23], that the account of operator H leads to

$$\frac{h_g^{(1)}(\vec{q}_1)}{h_g^{(0)}(\vec{q}_1)} \longrightarrow \omega^{(1)}(-\vec{q}_1^2) \left[\left(\frac{11}{6} - \frac{n_f}{3N} \right) + \left(\frac{11}{6} + \frac{(2+\epsilon)n_f}{6N} \right) - \frac{C_F n_f}{N^2} \left(\frac{2}{3} + \frac{\epsilon}{3} \right) \right]$$

$$\begin{aligned}
& -\frac{\epsilon}{N} \left(N \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5n_f}{9} \right) \\
& - g^2 N \vec{q}_1^2 \frac{1}{(2\pi)^{D-1}} \int \frac{d^{D-2}q_r}{\vec{q}_r^2 (\vec{q}_r - \vec{q}_1)^2} \ln \left(\frac{(\vec{q}_r - \vec{q}_1)^2}{\vec{q}_r^2} \right) \theta \left((\vec{q}_r - \vec{q}_1)^2 - \vec{q}_r^2 \right), \tag{7.15}
\end{aligned}$$

where $h_g^{(0)}$ and $h_g^{(1)}$ are notations of Ref. [23] for Born and one-loop parts of the forward helicity-conserving color singlet gluon impact factor. Note, $\Phi_G^{(0)(B)}$ and $h_g^{(0)}$ have different normalization, but comparing Eq. (2.5) of the present paper and Eq. (2.1) of Ref. [23], it is quite easy to see that the R.H.S. in (7.14) and (7.15) must coincide. Therefore we should check the equality

$$\begin{aligned}
& \int \frac{d^{D-2}q_r}{\vec{q}_r^2 (\vec{q}_r - \vec{q}_1)^2} \left[\ln \left(\frac{\vec{q}_r^2}{\vec{q}_1^2} \right) - \ln \left(\frac{\vec{q}_r^2}{(\vec{q}_r - \vec{q}_1)^2} \right) \theta \left((\vec{q}_r - \vec{q}_1)^2 - \vec{q}_r^2 \right) \right] = \\
& 2 \int \frac{d^{D-2}q_r}{\vec{q}_r^2 (\vec{q}_r - \vec{q}_1)^2} \ln \left(\frac{\vec{q}_r^2}{\vec{q}_1^2} \right) \theta \left(\vec{q}_r^2 - (\vec{q}_r - \vec{q}_1)^2 \right) = 0, \tag{7.16}
\end{aligned}$$

which must be satisfied with accuracy up to terms non-vanishing in the limit $\epsilon = 0$. As it is evident from second line of Eq. (7.16), the expression there has no poles in ϵ , therefore it is enough to check that

$$\begin{aligned}
I &= \int \frac{d^2q_r}{\vec{q}_r^2 (\vec{q}_r - \vec{q}_1)^2} \ln \left(\frac{\vec{q}_r^2}{\vec{q}_1^2} \right) \theta \left(\vec{q}_r^2 - (\vec{q}_r - \vec{q}_1)^2 \right) = \\
& \frac{1}{\vec{q}_1^2} \int \frac{d^2k}{\vec{k}^2 (\vec{k} - \vec{n})^2} \ln \vec{k}^2 \theta \left(\vec{k}^2 - (\vec{k} - \vec{n})^2 \right) = 0, \tag{7.17}
\end{aligned}$$

where k is a dimensionless vector and n an arbitrary vector with $\vec{n}^2 = 1$. Then one can make two changes of integration variables

$$\vec{k} \rightarrow \frac{\vec{k}}{\vec{k}^2}, \quad \vec{k} \rightarrow -(\vec{k} - \vec{n}) \tag{7.18}$$

to obtain

$$I = \frac{1}{\vec{q}_1^2} \int \frac{d^2k}{\vec{k}^2} \ln(\vec{k} - \vec{n})^2 \theta(1 - \vec{k}^2) = \frac{1}{\vec{q}_1^2} \int_0^{2\pi} d\phi \int_0^1 \frac{dk}{k} \ln(1 + k^2 - 2k \cos \phi). \tag{7.19}$$

As last step the integration by parts over k gives an expression which can be easily integrated over ϕ with zero result:

$$I = \frac{2\pi}{\vec{q}_1^2} \int_0^1 \frac{dk}{k} \ln k \left[\frac{(1 - k^2)}{2\pi} \int_0^{2\pi} \frac{d\phi}{(1 + k^2 - 2k \cos \phi)} - 1 \right] = 0. \tag{7.20}$$

8 Discussion

In this paper we have obtained the NLA non-forward gluon impact factors with any color structure in the t -channel in QCD with massive quarks at arbitrary space-time dimension

$D = 4 + 2\epsilon$. Then we have used the integral representation in the case of the octet impact factors to check the second bootstrap condition [14] and have found that it is satisfied. As it was mentioned above, this fact is very important from the theoretical point of view for the BFKL approach and demonstrates in very clear way the compatibility of this approach with the s -channel unitarity in the NLA. After this check we have carried out integrations in the form of an expansion in ϵ in the expression for the impact factor (2.9) in the important case of QCD with n_f massless quark flavors. For this last case the forward impact factor can be calculated exactly as a function of ϵ . As an example we have presented in Eq. (7.10) the singlet color helicity conserving impact factor. It was already obtained in Ref. [23] as an expansion in ϵ with the accuracy up to terms finite at $\epsilon \rightarrow 0$. We notice that, expanding our exact result (7.10), leads to an expression which is in agreement with the result of Ref. [23] when taking into account the differences in the definitions of impact factors.

Let us note, that throughout this paper we have used the unrenormalized coupling constant g , regularizing both ultraviolet and infrared divergences by the same parameter ϵ . The ultraviolet divergences are contained only in the one-gluon contribution and come from the one-loop correction to the gluon-gluon-Reggeon effective interaction vertex (see Section 3). This vertex was defined in Refs. [6], [20] and [21] in such way that the ultraviolet divergences can be removed by simple charge renormalization in the \overline{MS} scheme

$$g = g(\mu)\mu^{-\epsilon} \left[1 + \left(\frac{11}{3} - \frac{2}{3} \frac{n_f}{N} \right) \frac{g^2(\mu) N \Gamma(1 - \epsilon)}{2\epsilon(4\pi)^{2+\epsilon}} \right]. \quad (8.1)$$

After this renormalization has been performed there are still divergences of the infrared kind in the gluon impact factors because the gluon is not a colorless object. Note, that the definition of impact factors given in [14] and used in this paper guarantees the infrared safety of the impact factors for colorless particles [24].

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A Appendix A

In this section we calculate the integrals K_1 (see Eq. (5.21)), $K_2 - K_4$ and $K_{4\perp\perp}^{\mu\nu}$ (see Eq. (5.28)), appearing in the expressions (7.5) and (7.6) for quark-antiquark and two-gluon contributions to the gluon NLA non-forward impact factors (2.9) in completely massless QCD. Let us firstly consider K_1 . For the non-forward case we can present K_1 as follows:

$$K_1 = (\vec{q}^2)^\epsilon \widetilde{K}_1 ,$$

$$\widetilde{K}_1 = \frac{(4\pi)^{2+\epsilon}\Gamma(1+2\epsilon)}{4\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)} \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \ln\left(\frac{1}{\vec{k}^2}\right) \frac{1}{(\vec{k}-\vec{k}_1)^2(\vec{k}-\vec{k}_2)^2} , \quad (\text{A.1})$$

with

$$\vec{k}_1 = \frac{\vec{q}_1'}{|\vec{q}|} , \quad \vec{k}_2 = \frac{\vec{q}_1}{|\vec{q}|} , \quad (\vec{k}_1 - \vec{k}_2)^2 = 1 , \quad \vec{q} \neq 0 . \quad (\text{A.2})$$

We need for \widetilde{K}_1 an expression having accuracy up to terms of the type ϵ^0 and being exact in the regions $\vec{k}_1 \rightarrow 0$, $\vec{k}_2 \rightarrow 0$, according to the discussion after Eq. (7.6). With the help of the equality

$$\ln\left(\frac{1}{\vec{k}^2}\right) = \frac{d}{d\alpha} \left(\frac{1}{\vec{k}^2}\right)^\alpha \Big|_{\alpha=0} \quad (\text{A.3})$$

and of the generalized Feynman parametrization (see, for instance, Ref. [5]) the integration over \vec{k} gives

$$\begin{aligned} \widetilde{K}_1 &= \frac{\Gamma(1+2\epsilon)}{2\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)} \int_0^1 dz \left(\frac{\Gamma(1-\epsilon+\alpha)}{\Gamma(1+\alpha)} \alpha \right. \\ &\quad \left. \int_0^1 \frac{dx x^{\alpha-1} (1-x)^{\epsilon-\alpha}}{\left[x \left((1-z)\vec{k}_1^2 + z\vec{k}_2^2 \right) + (1-x)z(1-z) \right]^{1-\epsilon+\alpha}} \right)'_{\alpha=0} . \end{aligned} \quad (\text{A.4})$$

It is not difficult now to obtain a linear term in α in the expression inside the large brackets of the last relation in order to perform the differentiation. Then the integral \widetilde{K}_1 takes the form

$$\begin{aligned} \widetilde{K}_1 &= \frac{1}{\epsilon} \left(\frac{1}{\epsilon} + 2\psi(1+2\epsilon) - 2\psi(1+\epsilon) + \psi(1-\epsilon) - \psi(1) \right) \\ &+ \frac{\Gamma(1+2\epsilon)}{2\Gamma^2(1+\epsilon)} \left[\int_0^1 \int_0^1 dz dx \frac{\epsilon \ln x}{(1-x)} \frac{(1-x)^\epsilon}{\left[x \left((1-z)\vec{k}_1^2 + z\vec{k}_2^2 \right) + (1-x)z(1-z) \right]^{1-\epsilon}} \right. \\ &\quad \left. + \int_0^1 \int_0^1 dz dx \ln x \frac{(1-x)^\epsilon (1-\epsilon) \left((1-z)\vec{k}_1^2 + z\vec{k}_2^2 - z(1-z) \right)}{\left[x \left((1-z)\vec{k}_1^2 + z\vec{k}_2^2 \right) + (1-x)z(1-z) \right]^{2-\epsilon}} \right] . \end{aligned} \quad (\text{A.5})$$

This expression is still exact and holds for any ϵ . We should now perform the expansion in ϵ to carry out the integrations in Eq. (A.5) in an approximate form. As for the first integral in the R.H.S. of this equation, we find that it is order $O(\epsilon)$ and we neglect it in our calculation. In order to evaluate the second integral, we divide the region of integration over x in two parts in the following way:

$$1) \ 0 < x < \delta , \quad 2) \ \delta < x < 1 , \quad \delta \rightarrow 0 . \quad (\text{A.6})$$

In the first region the integration becomes simpler and gives without any difficulties the following contribution to the square brackets in the R.H.S. of Eq. (A.5):

$$-\frac{1}{\epsilon^2} \left((\vec{k}_1^2)^\epsilon + (\vec{k}_2^2)^\epsilon \right) + \ln^2 \delta , \quad (\text{A.7})$$

with accuracy up to ϵ^0 type terms. In the second region, we can evidently put $\epsilon = 0$, because the integration is convergent. The contribution of this integration region, analogous to that of Eq. (A.7), is

$$\begin{aligned} & -\ln^2 \delta - \ln \delta \ln(\delta \vec{k}_1^2 \vec{k}_2^2) + \int_0^1 dz \int_\delta^1 \frac{dx}{x} \frac{1}{\left[x \left((1-z) \vec{k}_1^2 + z \vec{k}_2^2 \right) + (1-x)z(1-z) \right]} \\ & = -\ln^2 \delta + \ln \vec{k}_1^2 \ln \vec{k}_2^2 . \end{aligned} \quad (\text{A.8})$$

As it must be, the dependence on δ is cancelled in the sum of the contributions (A.7) and (A.8), so that the limit $\delta \rightarrow 0$ does exist for this sum. Expanding with the required accuracy the first term and the coefficient in front of the square brackets in Eq. (A.5), we get

$$\begin{aligned} \frac{1}{\epsilon} \left(\frac{1}{\epsilon} + 2\psi(1+2\epsilon) - 2\psi(1+\epsilon) + \psi(1-\epsilon) - \psi(1) \right) &= \frac{1}{\epsilon^2} + \psi'(1) , \\ \frac{\Gamma(1+2\epsilon)}{2\Gamma^2(1+\epsilon)} &= \frac{1}{2} \left(1 + \epsilon^2 \psi'(1) \right) . \end{aligned} \quad (\text{A.9})$$

Using now Eqs. (A.7) and (A.8), for non-zero \vec{k}_1 or \vec{k}_2 (these vectors cannot be both zero at the same time, because $\vec{q} \neq 0$, according the definitions (A.2)) we obtain

$$\widetilde{K}_1 = -\frac{1}{2\epsilon} \ln(\vec{k}_1^2 \vec{k}_2^2) - \frac{1}{4} \ln^2 \left(\frac{\vec{k}_1^2}{\vec{k}_2^2} \right) . \quad (\text{A.10})$$

We should now include the correct asymptotics in the limits of small \vec{k}_1 or \vec{k}_2 . It is easy to check that the correct asymptotics of \widetilde{K}_1 in the limit, for example, $\vec{k}_1 \rightarrow 0$, is given by the expression

$$\begin{aligned} \widetilde{K}_1(\vec{k}_1 \rightarrow 0) &= \frac{(4\pi)^{2+\epsilon} \Gamma(1+2\epsilon)}{4\Gamma(1-\epsilon) \Gamma^2(1+\epsilon)} \left[(\vec{k}_1^2)^\epsilon \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \ln \left(\frac{1}{\vec{k}^2} \right) \frac{1}{(\vec{k} - \vec{n})^2} \right. \\ & \quad \left. + \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \ln \left(\frac{1}{\vec{k}^2} \right) \frac{1}{\vec{k}^2 (\vec{k} - \vec{n})^2} \right] , \end{aligned} \quad (\text{A.11})$$

where \vec{n} is an arbitrary vector with $\vec{n}^2 = 1$, the omitted terms being at least of order $|\vec{k}_1|$ in the region under consideration. The integrals in the R.H.S. of Eq. (A.11) can be easily calculated and, using also the symmetry of \widetilde{K}_1 under the replacement $\vec{k}_1 \leftrightarrow \vec{k}_2$, we arrive at the conclusion that the function

$$\frac{1}{2\epsilon^2} \left(2 - (\vec{k}_1^2)^\epsilon - (\vec{k}_2^2)^\epsilon \right) + \frac{1}{\epsilon} \left(\psi(1+2\epsilon) - \psi(1+\epsilon) + \psi(1-\epsilon) - \psi(1) \right) \quad (\text{A.12})$$

contains the correct asymptotics of \widetilde{K}_1 in both limits $\vec{k}_1, \vec{k}_2 \rightarrow 0$. To avoid double counting when including this asymptotics into the intermediate result (A.10), we should

add the function (A.12) in its exact form there and subtract it in the expression obtained by expanding in ϵ up to terms non-vanishing in the limit $\epsilon \rightarrow 0$. The final result for K_1 defined by Eqs. (A.1) and (A.2) takes the form

$$K_1 = \frac{1}{2} (\vec{q}^2)^\epsilon \left[\frac{1}{\epsilon^2} \left(2 - \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right)^\epsilon - \left(\frac{\vec{q}_1^2}{\vec{q}^2} \right)^\epsilon \right) + 4\psi''(1)\epsilon + \ln \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_1^2}{\vec{q}^2} \right) \right], \quad (\text{A.13})$$

where we did not expand in ϵ the common factor $(\vec{q}^2)^\epsilon$, because it would produce an additional complication, although such an expansion is possible in Eq. (A.13).

Our next step is the calculation of K_2 , defined in Eq. (5.28). It can be written in the following form:

$$K_2 = \left(-\frac{1}{2\epsilon} + \psi(1+2\epsilon) - \psi(1) \right) \left((\vec{q}_1^2)^\epsilon + (\vec{q}_1'^2)^\epsilon \right) + 2\epsilon \int_0^\infty dz \frac{\ln z}{(1+z)^{1+2\epsilon}} \frac{(\vec{q}(\vec{q}_1 + z\vec{q}_1'))}{((\vec{q}_1 + z\vec{q}_1')^2)^{1-\epsilon}}. \quad (\text{A.14})$$

In the region $q_1, q_1' \neq 0$ (here and everywhere below $k = |\vec{k}|$), when we can restrict ourselves to consider K_2 with accuracy up to terms linear in ϵ (see Eq. (7.6)), the integration in Eq. (A.14) is performed in a quite straightforward way and the final result takes the form

$$K_2 = -\frac{1}{\epsilon} - \ln(q_1 q_1') + 4\epsilon\psi'(1) - 2\epsilon \ln q_1 \ln q_1' - \epsilon\theta^2, \quad (\text{A.15})$$

with

$$\cos \theta = \frac{(\vec{q}_1 \vec{q}_1')}{q_1 q_1'}, \quad 0 < \theta < \pi. \quad (\text{A.16})$$

Unfortunately, the correct asymptotics of the R.H.S. of Eq. (A.14) in the limits $q_1 \rightarrow 0$ and $q_1' \rightarrow 0$ cannot be expressed in terms of elementary functions only, so that it seems to be better to leave the integral K_2 in the exact form of Eq. (A.14), which is convenient for subsequent applications of the gluon impact factors (2.9).

The remaining integrations of K_3 , K_4 and $K_{4\perp\perp}^{\mu\nu}$ (5.28) are easily performed and the results we obtain for the non-forward case $q \neq 0$, valid also in the regions of small q_1 or q_1' , are:

$$K_3 = 1 - 2\epsilon + \epsilon \ln(q_1 q_1') + \frac{\epsilon}{q^2} \left((q_1^2 - q_1'^2) \ln \left(\frac{q_1}{q_1'} \right) + 2q_1 q_1' \theta \sin \theta \right) + 2\epsilon^2 (2 - 2 \ln q + \ln^2 q), \quad (\text{A.17})$$

$$\begin{aligned} K_4 = & \frac{1}{6} - \frac{5\epsilon}{18} + \frac{\epsilon}{6} \ln(q_1 q_1') - \frac{2\epsilon q_1 q_1'}{3q^4} (2q_1 q_1' \sin^2 \theta - q^2 \cos \theta) + \frac{\epsilon(q_1^2 - q_1'^2)}{6q^6} (8q_1^2 q_1'^2 \sin^2 \theta \\ & - 2q_1 q_1' q^2 \cos \theta + q^4) \ln \left(\frac{q_1}{q_1'} \right) + \frac{2\epsilon q_1^2 q_1'^2}{3q^6} (4q_1 q_1' \sin^2 \theta - 3q^2 \cos \theta) \theta \sin \theta \\ & + \frac{\epsilon^2}{54} (19 - 30 \ln q + 18 \ln^2 q), \end{aligned} \quad (\text{A.18})$$

$$K_{4\perp\perp}^{\mu\nu} = \frac{(q_{1\perp}^{\mu} q_{1\perp}^{\nu} + q_{1\perp}^{\mu} q_{1\perp}^{\nu})}{2q_{1\perp}^2} \left[-1 + \frac{(q_1^2 - q_1'^2)}{q^2} \ln \left(\frac{q_1}{q_1'} \right) + (2q_1 q_1' \sin^2 \theta - q^2 \cos \theta) \frac{\theta}{q^2 \sin \theta} \right]$$

$$\begin{aligned}
& + \frac{(q_{1\perp}'^\mu q_{1\perp}'^\nu - q_{1\perp}^\mu q_{1\perp}^\nu)}{2q_{1\perp}^2} \left[-\frac{2(q_1^2 - q_1'^2)}{q^2} + \ln\left(\frac{q_1}{q_1'}\right) + \frac{2q_1 q_1'}{q^4} (3q^2 \cos\theta - 4q_1 q_1' \sin^2\theta) \ln\left(\frac{q_1}{q_1'}\right) \right. \\
& \quad \left. + (4q_1 q_1' \sin^2\theta - q^2 \cos\theta) \frac{(q_1^2 - q_1'^2)\theta}{q^4 \sin\theta} \right] - \frac{q_{1\perp}^\mu q_{1\perp}^\nu}{q_{1\perp}^2} \left[\frac{1}{3} + \frac{\epsilon}{18} (5 - 6 \ln q) \right. \\
& \quad \left. + \frac{q_1 q_1'}{q^4} (3q^2 \cos\theta - 4q_1 q_1' \sin^2\theta) + \frac{2q_1 q_1'}{q^6} (q_1^2 - q_1'^2) (2q_1 q_1' \sin^2\theta - q^2 \cos\theta) \ln\left(\frac{q_1}{q_1'}\right) \right. \\
& \quad \left. + (q^4 - 2(q_1^2 - q_1'^2)^2 \sin^2\theta) \frac{q_1 q_1' \theta}{q^6 \sin\theta} \right]. \tag{A.19}
\end{aligned}$$

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